Online Appendix: The Macroeconomics of International Remittance Flows

A Data Appendix: Publicly Available Data

A.1 Risk-Sharing Correlations

Table A.1: First Difference Correlation Matrix

	OECD		World Bank	
	1970-2001	2002-2024	1970-2001	2002-2024
Austria	-0.14	0.17	-0.12	0.39
Canada	-0.26	-0.15	-0.11	0.31
Colombia	-0.66	-0.10		0.04
Denmark	-0.20	-0.20	-0.20	0.18
Finland	-0.51	-0.43	-0.17	-0.02
France	-0.18	0.13	-0.15	0.43
Germany	-0.33	0.51	-0.39	0.55
Greece	-0.16	-0.24	-0.36	-0.07
Ireland	0.04	-0.15	0.19	0.13
Italy	-0.25	0.15	-0.09	0.42
Japan	-0.08	0.09	-0.16	0.03
Mexico	-0.09	-0.52		-0.45
Netherlands	-0.19	0.19	-0.32	0.40
Portugal	-0.46	0.15	-0.14	0.42
Spain	-0.44	-0.15	-0.17	0.17
Sweden	-0.28	0.21	-0.18	0.40
United Kingdom	-0.36	0.05	-0.03	0.23
Bangladesh				-0.16
China				-0.48
India				0.07
Nepal				0.21
Pakistan				0.31
Philippines				0.35
Turkiye				0.07

Data: World Bank, 1970 - 2024 & OECD, 1970 - 2024. Each column correlates the real exchange rate with consumption relative to the U.S. (corr$(\Delta(c_t - c_t^{US}), \Delta q_t)$). The first two columns use OECD's average exchange rate and final consumption expenditure chain-linked data. The latter two columns use World Bank's exchange rate and final consumption expenditure. All calculations used data expressed in 2015 real USD.

Table A.2: World Bank - Remittance Consumption Correlation Matrix

	corr$(\Delta\omega_t^*, \Delta(c_t - c_t^{US}))$	corr$(\Delta q_t, \Delta(c_t - c_t^{US}))$	corr$(\Delta\omega_t^*, \Delta q_t)$
Bangladesh	-0.19	-0.16	0.32
China	-0.16	-0.48	-0.04
Colombia	-0.21	0.04	0.79
India	0.07	0.07	0.31
Mexico	-0.41	-0.45	0.57
Nepal	-0.01	0.21	0.41
Pakistan	-0.43	0.31	-0.21
Philippines	0.26	0.35	0.69
Turkiye	0.09	0.07	0.35

Data: World Bank, 2002 - 2024. First-difference correlation matrix. Corr$(\Delta\omega_t^*, \Delta(c_t - c_t^{US}))$ is change in remittances correlated to change in consumption relative to the U.S. Corr$(\Delta q_t, \Delta(c_t - c_t^{US}))$ is change in exchange rate correlated to change in consumption relative to the U.S. Corr$(\Delta\omega_t^*, \Delta q_t)$ is change in remittances correlated to change in exchange rate. All calculations used data expressed in 2015 real USD.

Table A.3: World Bank - Remittance GDP Correlation Matrix

	corr$(\Delta\omega_t^*, \Delta(y_t - y_t^{US}))$	corr$(\Delta q_t, \Delta(y_t - y_t^{US}))$	corr$(\Delta\omega_t^*, \Delta q_t)$
Bangladesh	-0.08	-0.25	0.32
China	0.14	-0.63	-0.04
Colombia	-0.2	-0.15	0.79
India	-0.12	-0.15	0.31
Mexico	-0.27	-0.39	0.57
Nepal	-0.17	0.01	0.41
Nigeria	-0.08	-0.26	0.20
Pakistan	-0.03	0.02	-0.21
Philippines	0.39	0.21	0.69
Turkiye	0.13	-0.07	0.35

Data: World Bank, 2002 - 2024. First-difference correlation matrix. Corr$(\Delta\omega_t^*, \Delta(y_t - y_t^{US}))$ is change in remittances correlated to change in GDP relative to the U.S. Corr$(\Delta q_t, \Delta(y_t - y_t^{US}))$ is change in exchange rate correlated to change in GDP relative to the U.S. Corr$(\Delta\omega_t^*, \Delta q_t)$ is change in remittances correlated to change in exchange rate. All calculations used data expressed in 2015 real USD.

Remittance correlations. Table A.1 shows that the correlation between the real exchange rate and remittance flows is more positive than the correlation between the real exchange rate and relative GDP or consumption. A standard model with flexible remittances predicts a negative correlation between remittance flows and relative GDP, controlling for relative exchange rates. That is, remittance flows should increase when the sender country is performing relatively better than the recipient country. Moreover, the higher the share of the population that participate in remittance flows, the more negative the correlation.

Furthermore, a standard model also predicts a positive correlation between remittance flows and the real exchange rate conditional on relative output. That is, when the exchange rate of the recipient country depreciates relative to the sender country, Similarly, the higher the share of the population

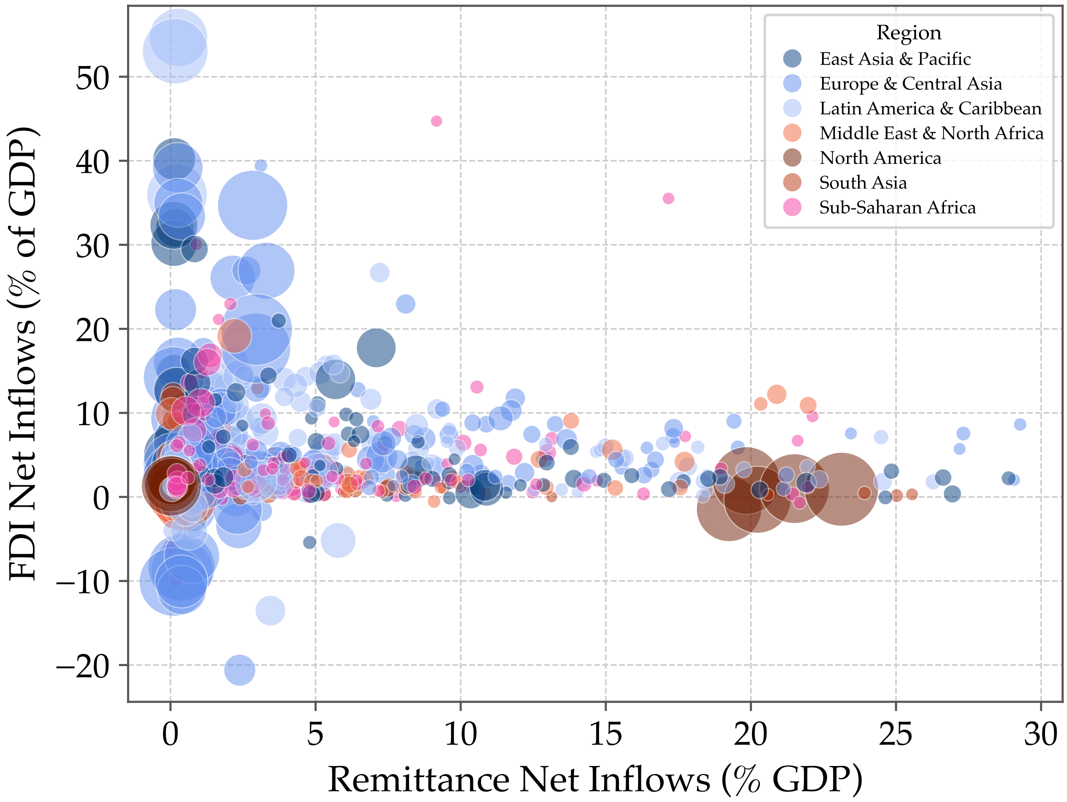
Table A.4: World Bank - Inflows

	Remittance Inflows (% GDP)	FDI Net Inflows (% GDP)
Bangladesh	7.0	0.8
Brazil	0.2	3.1
China	0.2	2.6
Colombia	2.1	3.9
India	3.2	1.6
Indonesia	1.0	1.7
Mexico	2.7	2.6
Nepal	21.3	0.3
Nigeria	4.6	1.3
Pakistan	5.7	1.0
Philippines	10.3	1.8
Russia	0.4	1.8
Turkiye	0.2	1.5

Data: World Bank, 2002 - 2024. Mean across listed countries from 2002-2024 remittances and FDI data, both as percentages of GDP.

that receives remittances, the more positive the correlation.

Figure A.1: Remittances and FDI Inflows Across Countries, 2002–2024



Notes: Data source — World Bank World Development Indicators (personal remittances received, % of GDP; foreign direct investment net inflows, % of GDP; GDP per capita, current USD), authors' calculations. Each marker is one country's average over a five-year period (2002–2006, 2007–2011, 2012–2016, 2017–2021, and 2022–2024, the last covering three years). Marker colour denotes the World Bank region and marker size is proportional to GDP per capita. World Bank regional and income-group aggregates are excluded. The sample is truncated to country-periods with remittances of at most 30% of GDP and FDI between −50% and 100% of GDP.

A.2 Survey Data

A.2.1 The Philippines

Table A.5: Summary Statistics - Survey of Overseas Filipinos

Destination Country	*Count*	*Percentage*
Saudi Arabia	5227	21.7
UAE	2967	12.3
Americas	1888	7.8
Europe	1885	7.8
Hong Kong	1770	7.4
Kuwait	1482	6.2
Singapore	1411	5.9
Qatar	1265	5.3
Japan	1100	4.6
Taiwan	1015	4.2
Other East Asia	900	3.7
Other Western Asia	882	3.7
Australia	759	3.2
Other SE Asia	635	2.6
Malaysia	574	2.4
Africa	276	1.1
Other	30	0.1
Total	24066	100.0

Notes: Data source — Survey of Overseas Filipinos (2018-2023). Counts are the total number of Filipino emigrants in each country.

Table A.6: Summary Statistics - Survey of Overseas Filipinos

Variable	*Mean*	*Median*
Age	37.9	36.0
Female	0.59	
Married	0.61	
Undergraduate Degree	0.55	
Graduate Degree	0.37	
Blue Collar Worker	0.83	
Monthly Average Income	714	466
Remittance Sent (past 6 mos.)	991	570

Notes: Data source — Survey of Overseas Filipinos (2018-2023). Income and remittances are in real USD. Undergraduate degree and graduate degree consists of those who completed the respective grade.

A.2.2 Bangladesh

Table A.7: Summary Statistics - Bangladesh Integrated Household Survey

Demographics	*Household Head*	*Migrant Non-Remitter*	*Migrant Remitter*	*Remittance Sender*
Age	45.7	24.3	30.3	.
Female	0.19	0.55	0.07	.
Undergraduate Degree	0.01	0.04	0.03	.
Graduate Degree	0.005	0.011	0.014	.
Blue Collar Worker	0.94	0.58	0.44	.
Married	0.89	.	.	.
Annual Income Incl Remit	2286	.	.	.
Annual Income Excl Remit	1451	.	.	.
Remittances Out (past 12 mos.)	557	.	.	.
Remittances In (past 12 mos.)	1428	.	1124	1249
Migrants in Household (avg)	1.2	.	.	.
Total Persons	4282	1002	1503	1930

Notes: Data source — Bangladesh Integrated Household Survey. Income and remittances are in real USD. Annual income including remittances only includes the incomes of those receiving remittances. Migrants were household members in the past baseline survey and are currently living away for 6 months or more in a different upazilla or country. Of those migrants, Migrant Non-Remitters did not send remittances and Migrant Remitters sent remittances to their household. Remittance Senders are migrants and those who were never household members who sent remittances to the household.

A.2.3 Mexico

Table A.8: Summary Statistics - National Survey of Household Income and Expenditure

Demographics	Households Not Receiving Remittances	Households Receiving Remittances
Age	49.6	52.9
Female	0.3	0.4
Undergraduate Degree	0.14	0.04
Graduate Degree	0.03	0.01
Total Household Members	4	4
Children $<$ 12	1	1
Average Monthly Income Excl Remit	3968	2895
Average Monthly Income Incl Remit	.	3088
Average Monthly Remittances	.	673
Savings	230	154
Savings (deposits only)	113	80
Total Households	353329	22134

Notes: Data source: National Survey of Household Income and Expenditure. Income and remittances are in real 2018 USD. Income is the average monthly total of all earning household members. Annual income, including remittances, only includes the income of those receiving remittances. Savings are all financial expenditures minus financial income. Savings (deposits only) is savings deposits minus withdrawals.

B Data Appendix: MTO Data

Table B.1: Countries in the Sample - 2014-2023.

Sender Countries	
	Australia, Austria, Bahrain, Belgium, Benin, Brazil, Bulgaria, Burkina Faso, Cameroon, Canada, Côte d'Ivoire (Ivory Coast), Croatia, Cyprus, Czechia (Czech Republic), Denmark, Estonia, Finland, France, Germany, Ghana, Gibraltar, Greece, Hong Kong, Hungary, Iceland, Ireland, Italy, Japan, Jordan, Kuwait, Latvia, Lithuania, Luxembourg, Malaysia, Malta, Netherlands, New Zealand, Norway, Oman, Philippines, Poland, Portugal, Qatar, Romania, Rwanda, Saudi Arabia, Senegal, Singapore, Slovakia, Slovenia, South Africa, South Korea (Republic of Korea), South Sudan, Spain, Sweden, Switzerland, Taiwan, Tanzania, Togo, Uganda, United Arab Emirates, United Kingdom, U.S., Zimbabwe.
Receiver Countries	
	Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Armenia, Australia, Austria, Bahrain, Bangladesh, Barbados, Belarus, Belgium, Benin, Bolivia, Bosnia and Herzegovina, Botswana, Brazil, British Virgin Islands, Bulgaria, Burkina Faso, Burundi, Cabo Verde (Cape Verde), Cambodia, Cameroon, Canada, Cayman Islands, Central African Republic, Chad, Chile, China, Colombia, Comoros, Costa Rica, Côte d'Ivoire (Ivory Coast), Croatia, Cuba, Cyprus, Czechia (Czech Republic), Democratic Republic of the Congo, Denmark, Dominican Republic, Ecuador, Egypt, El Salvador, Estonia, Ethiopia, Fiji, Finland, France, Gabon, Georgia, Germany, Ghana, Greece, Grenada, Guatemala, Guinea, Guinea-Bissau, Haiti, Honduras, Hong Kong, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Jamaica, Jordan, Kazakhstan, Kenya, Kuwait, Kyrgyzstan, Laos (Lao PDR), Latvia, Lebanon, Lesotho, Liberia, Lithuania, Madagascar, Malawi, Malaysia, Maldives, Maldives, Mali, Malta, Mauritania, Mauritius, Mexico, Moldova, Monaco, Morocco, Mozambique, Namibia, Nepal, Netherlands, Netherlands Antille, New Zealand, Nicaragua, Niger, Nigeria, Norway, Oman, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Qatar, Republic of the Congo, Romania, Russia, Rwanda, Saint Kitts and Nevis, Saint Lucia, Saint Vincent and the Grenadines, Samoa, Senegal, Seychelles, Sierra Leone, Singapore, Slovakia, Slovenia, Somalia, South Africa, South Korea (Republic of Korea), South Sudan, Spain, Sri Lanka, Suriname, Sweden, Switzerland, Tajikistan, Tanzania, Thailand, The Gambia, Togo, Tonga, Tunisia, Turkey, Turks and Caicos Islands, Uganda, Ukraine, United Arab Emirates, United Kingdom, United States.

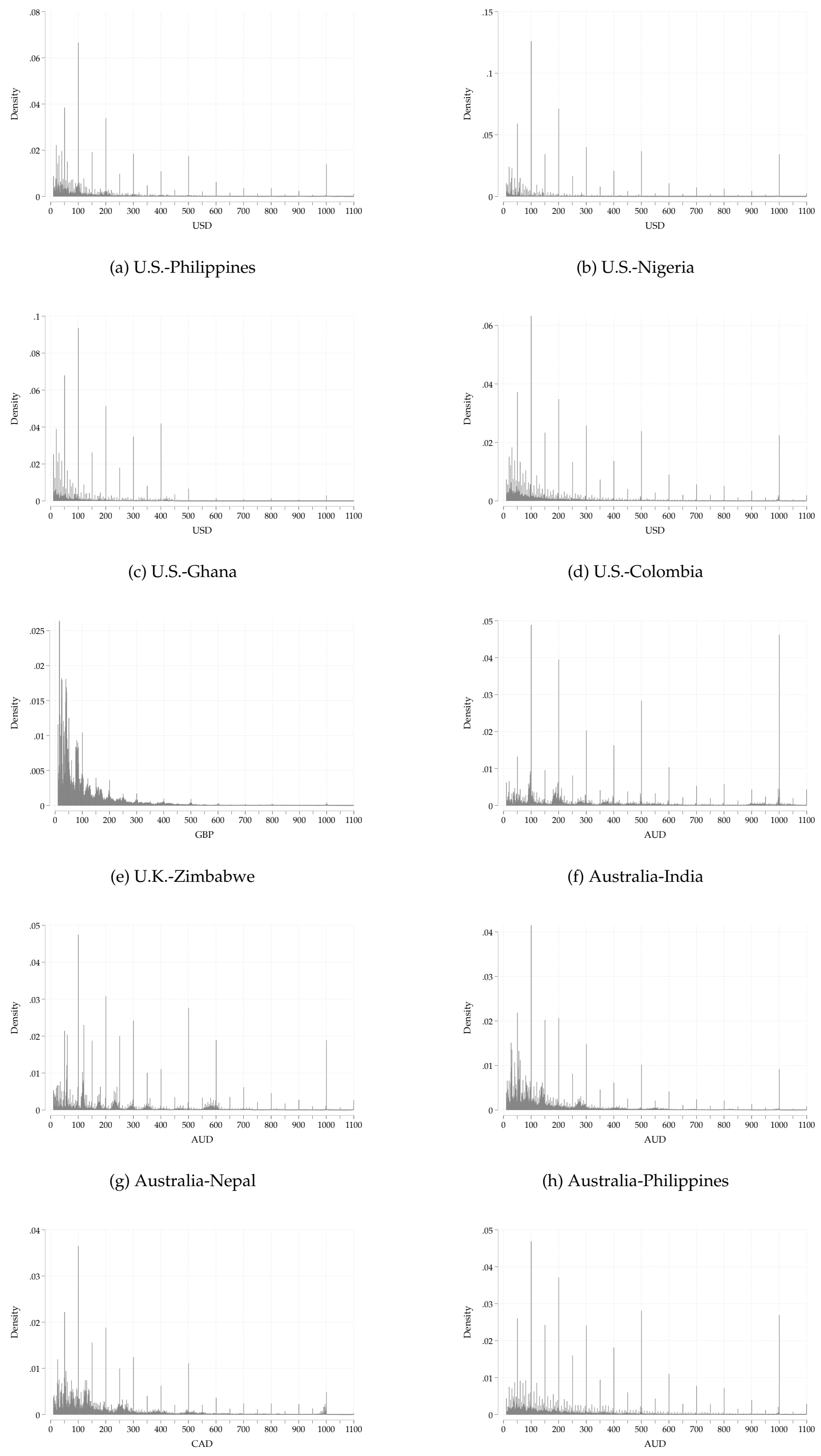
Table B.2: Summary Statistics: 2014–2018

		Total				Transaction					per Sender/Year		per Dyad/Year	
Corridor	**Female**	**Trans.**	**Volume**	**Senders**	**Dyads**	**Mean**	**Median**	**St. Dev.**	**Avg Fee ($)**	**Avg Fee %**	**Trans.**	**Avg. Days**	**Trans.**	**Avg. Days**
U.S. – Philippines	33%	896.1k	167m	46.4k	155k	186	90	326	3.5	5.8%	14	8	5	16
U.S. – Nigeria	45%	984.0k	199m	52.6k	273.4k	202	80	423	5.3	8.5%	14	10	3	24
Australia – India	32%	793.9k	578m	48.8k	167.7k	728	360	1099	2.3	2.3%	8	23	3	35
United Kingdom – Zimbabwe	59%	4.8m	183m	38.2k	308.2k	126	52	272	4.3	6.5%	16	11	3	28
Australia – Nepal	29%	25.9k	821m	3.2k	8.3k	176	12	585	2.4	1.8%	6	19	33	38
U.S.– Ghana	34%	619.0k	105m	40.8k	163.9k	170	80	341	3.3	4%	12	9	3	22
Australia – Philippines	40%	1.5m	262m	28.1k	184.5k	169	82	312	4.6	6.3%	26	8	6	16
Canada – Philippines	57%	1m	159m	36.3k	173.5k	164	99	190	5.6	6.7%	14	13	4	24
U.S.– Colombia	53%	121.6k	42m	11.0k	25k	350	160	499	2.7	1.7%	9	14	4	20
Australia– Colombia	50%	302.3k	113m	16.8k	430k	375	211	499	3	4.4%	10	19	5	26

Notes: Trans. = Transactions; Avg. Days = Average days between transactions. All the volumes are in USD.

B.1 The Reference Currency of Remittance Senders

Figure B.1: Amount Sent in Sender Currency



(a) U.S.-Philippines

(b) U.S.-Nigeria

(c) U.S.-Ghana

(d) U.S.-Colombia

(e) U.K.-Zimbabwe

(f) Australia-India

(g) Australia-Nepal

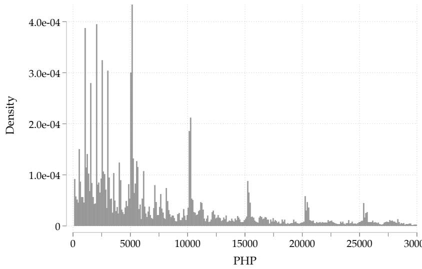
(h) Australia-Philippines

(i) Canada-Philippines

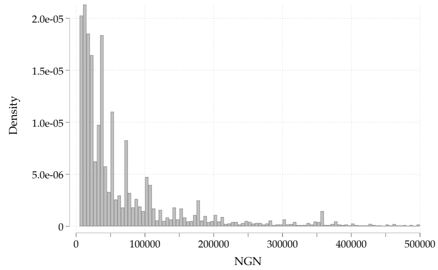
(j) Australia-Colombia

Notes: MTO data. Author's calculations. In the United Kingdom-Zimbabwe corridor, the only receiving currency available is the USD.

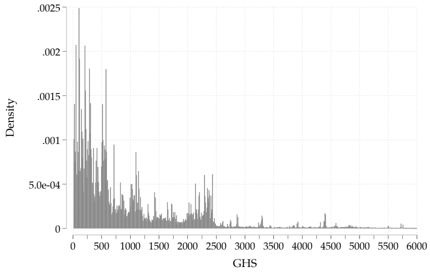
Figure B.2: Amount Sent in Receiver Currency



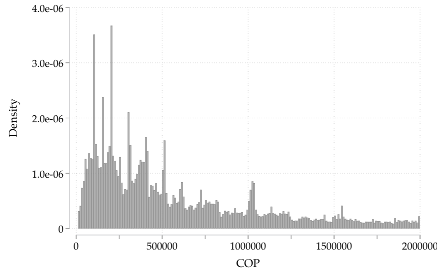
(a) U.S.-Philippines



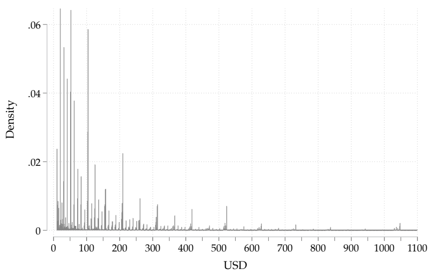
(b) U.S.-Nigeria



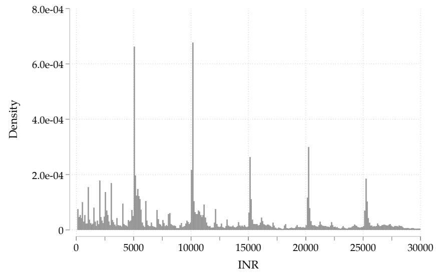
(c) U.S.-Ghana



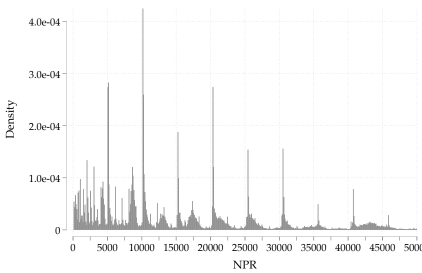
(d) U.S.-Colombia



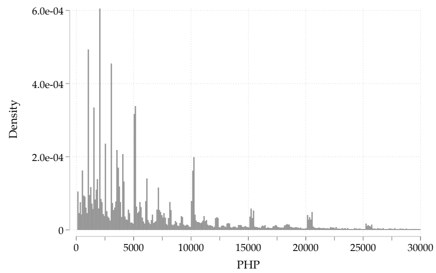
(e) U.K.-Zimbabwe



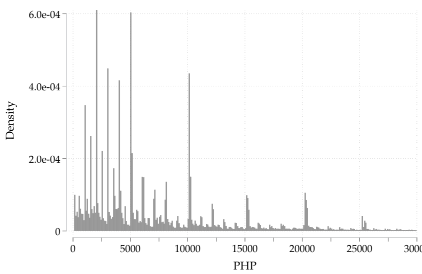
(f) Australia-India



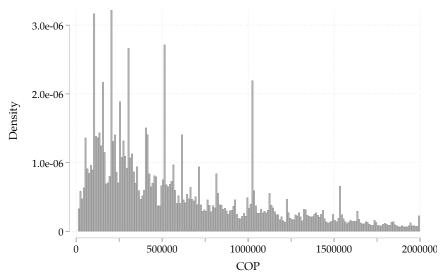
(g) Australia-Nepal



(h) Australia-Philippines



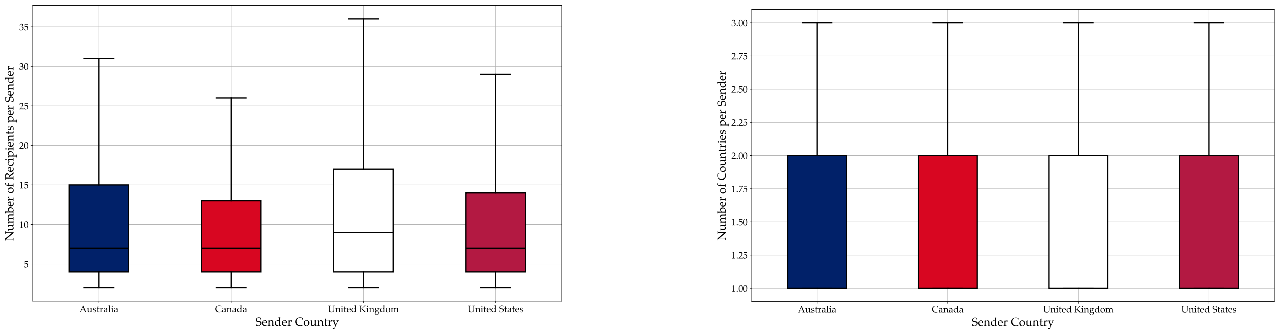
(i) Canada-Philippines



(j) Australia-Colombia

Notes: MTO data. Author's calculations. In the United Kingdom-Zimbabwe corridor, the only receiving currency available is the U.S. dollar.

Figure B.3: Recipients and Countries Sent per Sender



(a) Recipient Accounts per Sender

(b) Countries per Sender

Notes: MTO data. Author's calculations.

B.2 The Network of Senders and Recipients

How many recipients depend on each sender? How many remittance corridors does each sender participate in? To answer these questions, we focus on individuals based in the four largest sending countries, which are Australia, Canada, the United Kingdom and the U.S. Figure B.3a shows the distribution of the number of accounts that each sender sends to. We've restricted the sample of recipients to include only those who have received at least three transactions from a given sender.[19] On average, senders in the United Kingdom send to more accounts than those in Australia, which in turn has sent to more accounts than those in the U.S. and Canada. The median number of recipients ranges from 7 in the U.S. and Canada, to 9 in the United Kingdom.

Conversely, we find that on average each recipient account only receives remittance inflows from one sender account (Figure B.3b). Note that as mentioned in section 2.1, we have much more limited information regarding recipients. However, even if recipients have multiple accounts, they do not appear to receive remittance inflows from the same sender account. This suggests that the network of recipients is concentrated on a given sender as the given sender sends remittance flows to multiple recipient accounts.

What do we know about how many remittance corridors each sender participates in? Figure B.3b shows the distribution of the number of countries each sender sends remittances. We find that each sender sends to between 1 and 3 countries, with the majority sending to just one country. Overall, this suggests that the network of senders and recipients is sparse across remittance corridors but is more concentrated within a corridor.

[19] We do this to exclude one-off transactions sent through the MTO.

B.3 The prominence of the U.S. Dollar

In the majority of remittance corridors, the received currency is the local currency of the receiving country. The main reason for this is that it's usually the only currency offered by the MTO.[20] This accounts for around 86% of all transactions. Of the remaining transactions, almost all are received in U.S. dollars.[21] Figure B.4 shows the most prominent instances of the U.S. dollar being used as the received currency despite neither the sender's nor recipient's countries being the U.S. These include Nigeria, Turkey, Russia and Uruguay. These represent corridors in which more than one currency is available in the sender's choice set of receiving currencies.

The figure shows the sender countries on the left and the recipient countries on the right. The links represent the remittance flows and the colour of the links represent the received currency. Red flows represent the flows received in U.S. dollars. Notice that there are prominent red flows from non-U.S. senders towards recipient countries. The blue links represent flows where the receiving currency is Euros and all other colours represent flows where the receiving currency is the recipient country's local currency. The main takeaway is that the U.S. dollar is used as a receiving currency for remittance flows even in transactions not involving the U.S.

In Turkey, 81% of transactions are received in U.S. dollars and the remainder in Euros (18%) and Turkish Lira (1%). Of the transactions received in U.S. dollars, only 44% are sent from the U.S. with the remainder from outside the U.S. The Euro transactions are mostly sent by Germany, Netherlands, Ireland, Finland, France and Belgium. A similar pattern features in Russia as only 1% of the transactions are received in Rubles. Notably, 62% of the transactions are received in U.S. dollars and around 50% of these transactions come from non-US countries. In particular Romania, Poland, Czechia and Hungary. The remaining 37% is performed in Euros and mostly sent from Germany, France, the Netherlands and Norway.

In Nigeria around 27% of the transactions are received in U.S. dollars. Of these, 67.5% come from the U.S., 10% from the United Kingdom, and 4.5% from Canada. Note that for Nigeria, the share of transactions received in U.S. dollars has fluctuated over time. This is partly due to the policy of the Central Bank of Nigeria on allowing foreign currency transactions within Nigeria. In Uruguay, 52% of transactions are received in U.S. dollars and 40% of those come from New Zealand. Only 7.5% are from the U.S.[22]

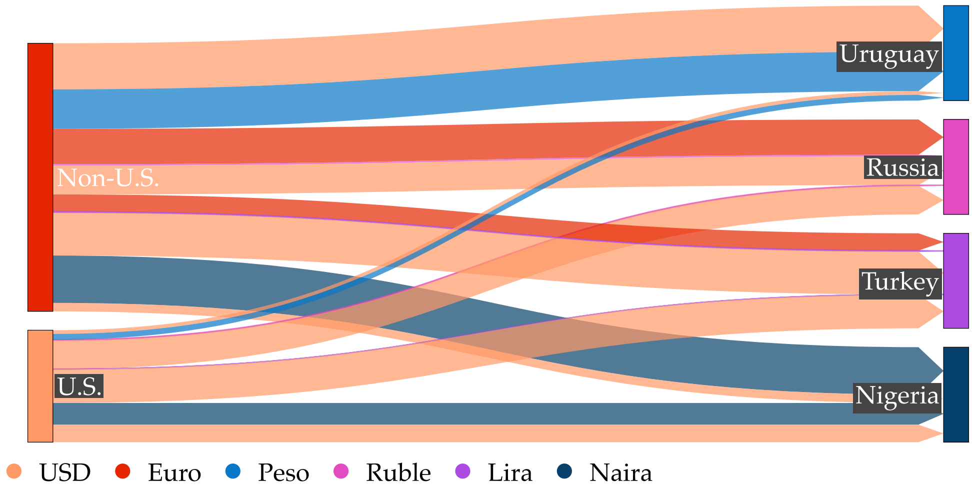
In summary, though the receiver's local currency is usually the most common receiving currency, there exist corridors for which the U.S. dollar is a receiving currency. This reflects a combination of demand for U.S. dollars in recipient countries, the ease with which the MTO has partners that can facilitate transactions in U.S. dollars and monetary policy.

[20] For example, in Zimbabwe 100% of the transactions are received in the U.S. dollar. This is due to the fact that the MTO only offers the possibility to send U.S. dollars. For Argentina, 100% of the transactions are received in Argentine Pesos. Similarly, this is due to the fact that MTO only allows transactions in Argentine Pesos to Argentina.

[21] Almost all of these are received in U.S. dollars; the small remainder is in euros and originates within the European Economic Area.

[22] The U.S. dollar is a prominent receiving currency in several further corridors spanning Southeast Asia, East Africa and Latin America. In each case a majority of transactions are dollar-denominated, whilst only a minority originate in the U.S.

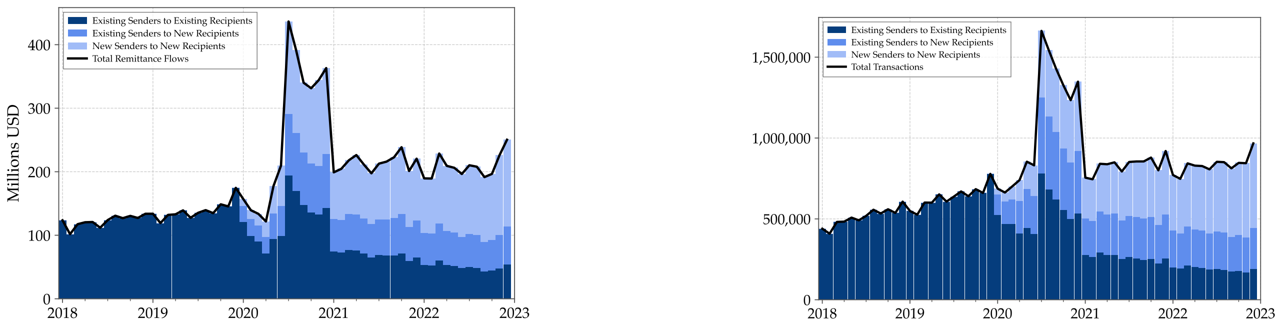
Figure B.4: Selected Remittance Flows by Currency



Notes: MTO data. Author's calculations.

B.4 Customer Base: Extensive and Intensive Margins

Figure B.5: Remittances by Tenure of Senders and Recipients

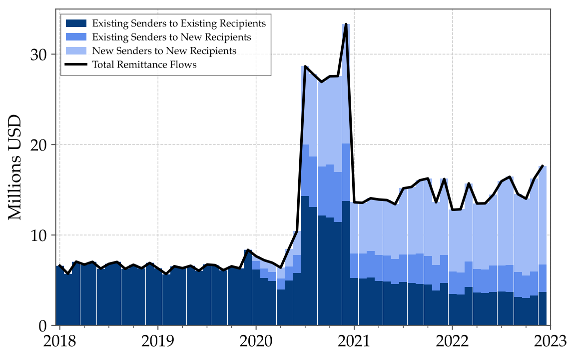


(a) Volume, USD

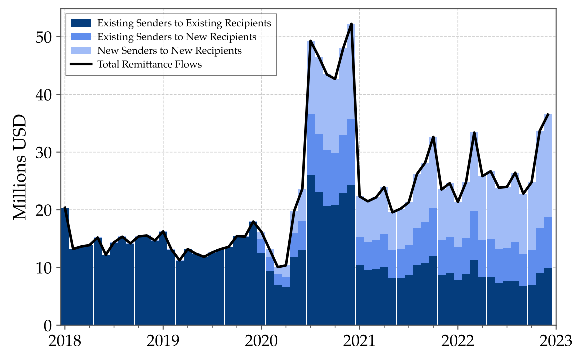
(b) Transactions

Notes: MTO data. Author's calculations. 'New Sender' refers to MTO's customer who signed up after January 1, 2020. 'New Receivers' refers to receivers whose ID appeared for the first time after January 1, 2020.

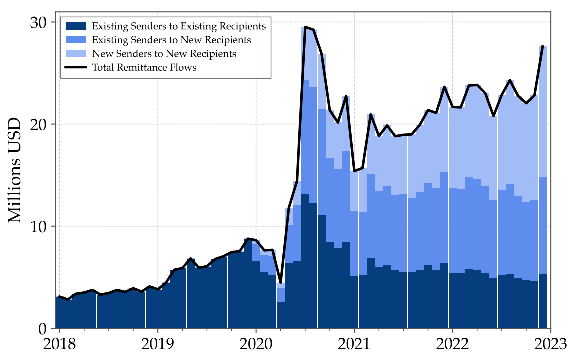
Figure B.6: Volume of Remittance Flows during the Pandemic by Tenure of Customers, USD



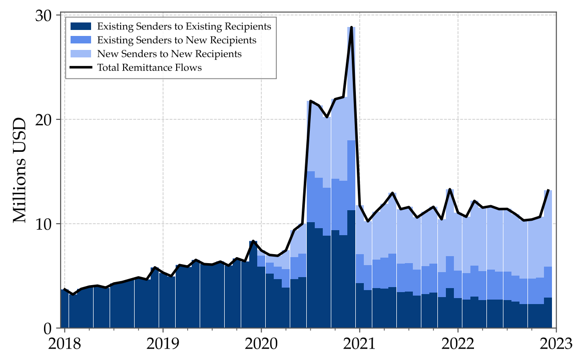
(a) Australia-Philippines



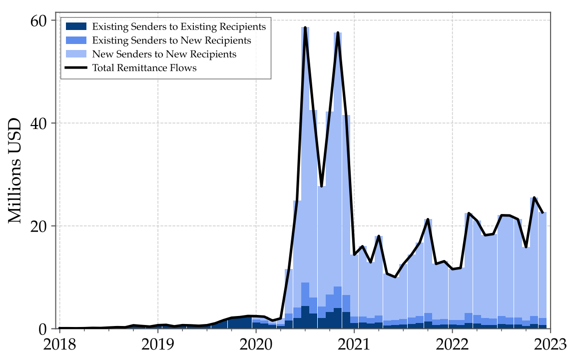
(b) Australia-India



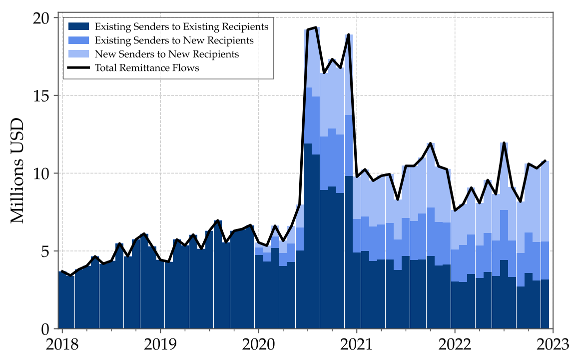
(c) United Kingdom – Zimbabwe



(d) Canada – Philippines

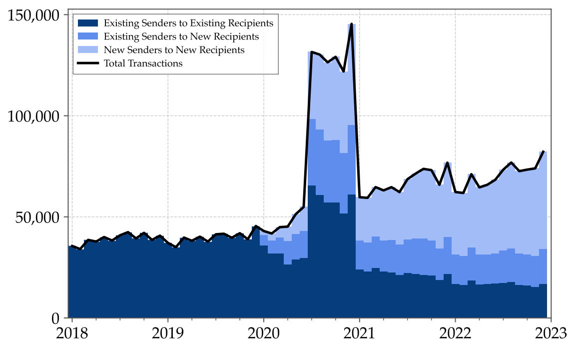


(e) Australia – Nepal

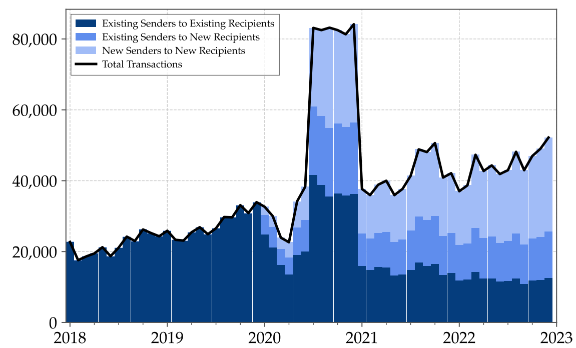


(f) Australia – Colombia

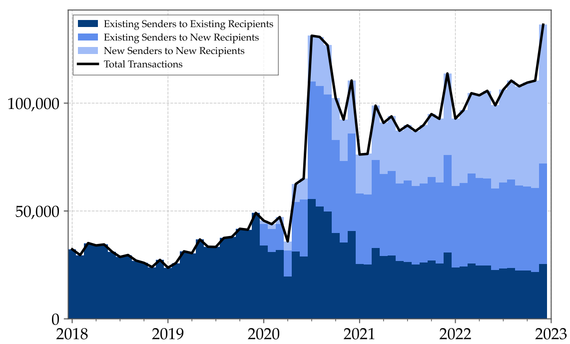
Figure B.7: Number of Transactions during the Pandemic by Tenure of Customers



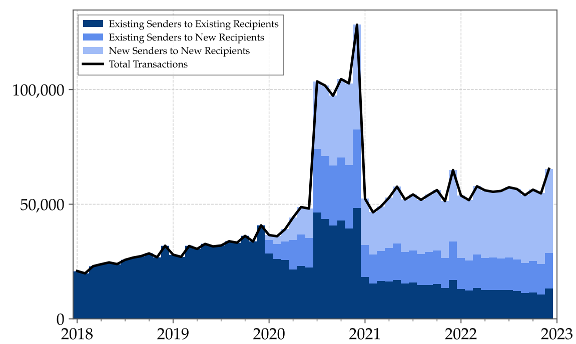
(a) Australia – Philippines



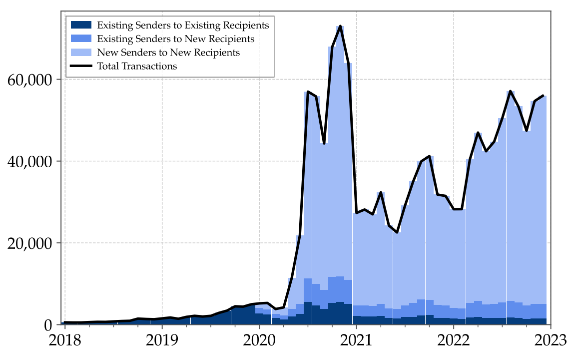
(b) Australia – India



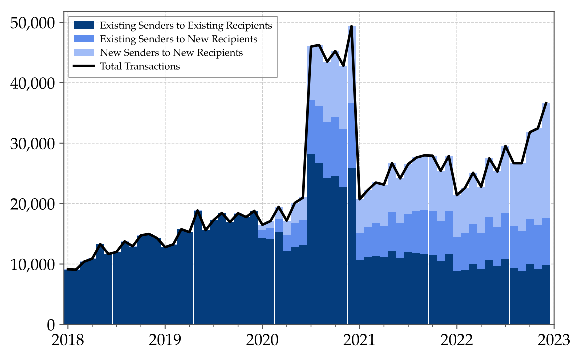
(c) United Kingdom – Zimbabwe



(d) Canada – Philippines

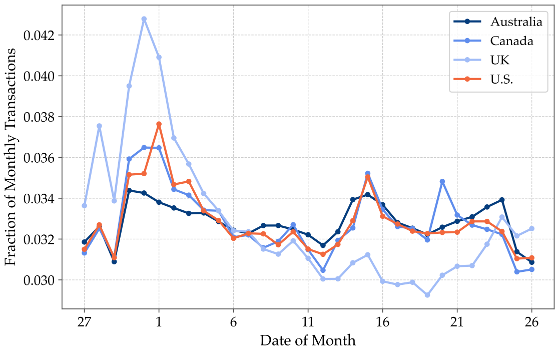


(e) Australia – Nepal

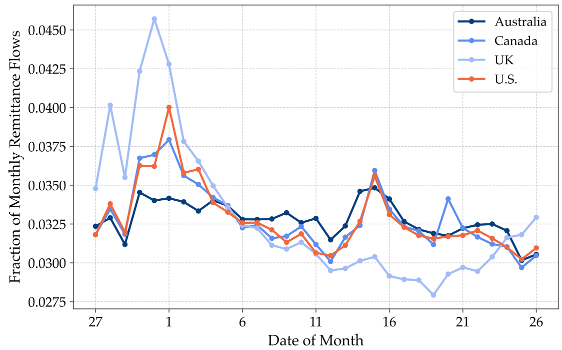


(f) Australia – Colombia

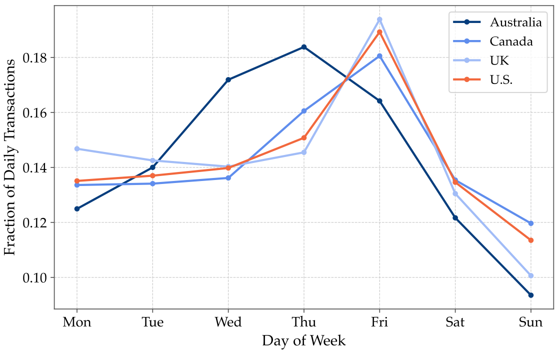
Figure B.8: Fraction of Transactions and Volume by Day of Month



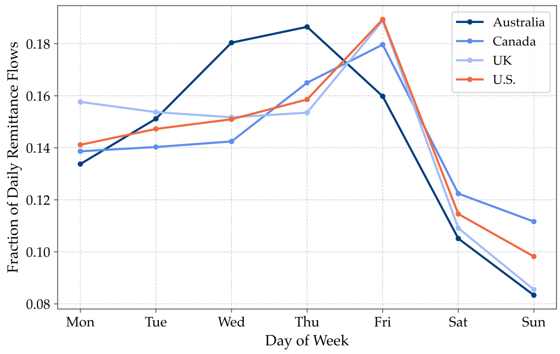
(a) Transactions by Date of Month



(b) Volume by Date of Month



(c) Transactions by Day of Week

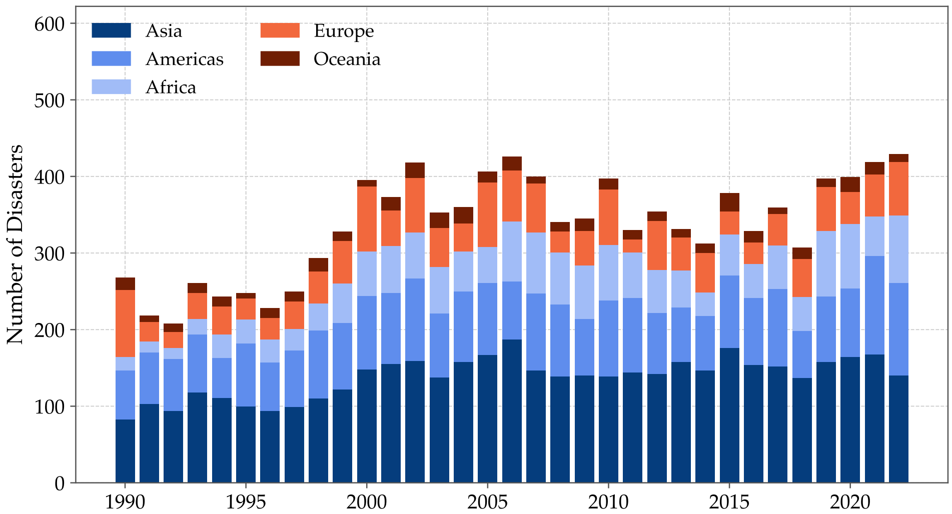


(d) Volume by Day of Week

Notes: MTO data. Author’s calculations. Transactions are dated in the sender country’s local time.

B.5 Natural Disasters

Figure B.9: Number of Natural Disasters by Region, 1990-2022



Notes: Data source — EM-DAT, CRED/UCLouvain. Author's calculations. The figure reports annual counts of natural disasters from 1990 to 2022, grouped by affected region. Counts are based on EM-DAT event records for disasters classified as natural, including weather-related and geophysical events. Biological events are excluded. Events are not weighted by fatalities, damages, or the number of people affected.

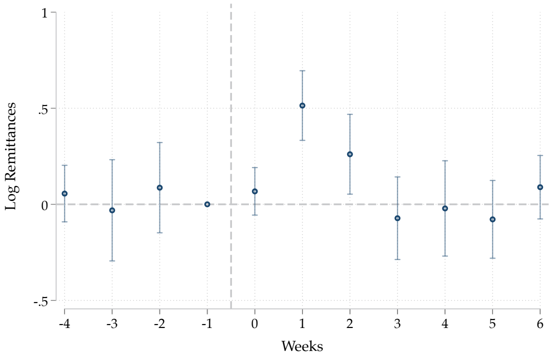
Table B.3: Natural Disaster Persistence

Country	Storm	Storm duration	Intensity	Deaths	Affected	Damage
India	Cyclone Yaas (26 May 2021)	23–28 May;	∼140 km/h	∼20	∼1.3M	∼$3.0bn
Philippines	Typhoon Rai/Odette (16 Dec 2021)	12–21 Dec	∼195 km/h	409	∼16M	∼$951M
Bangladesh	Cyclone Sitrang (24 Oct 2022)	20–25 Oct	∼88 km/h (mild)	16–35	∼2.3M	Modest

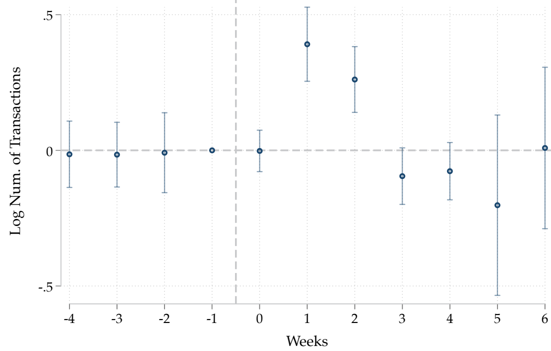
Notes: Sources include Wikipedia, UN-OCHA and UNICEF situation reports, and news agencies. Figures should be cross-checked against EM-DAT for citation.

B.5.1 The Philippines

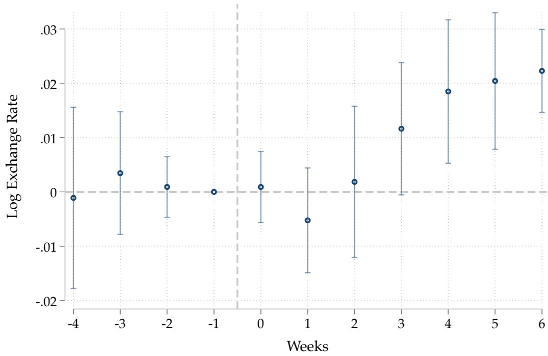
Figure B.10: Event Study: Typhoon ”Rai” (also known as ”Odette”) in the Philippines



(a) Log Amount Sent in USD.



(b) Log Total Transactions.

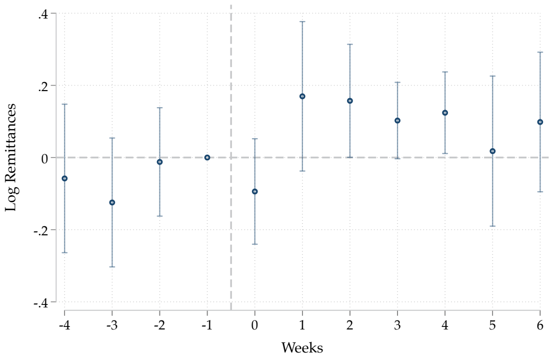


(c) Log Exchange Rate

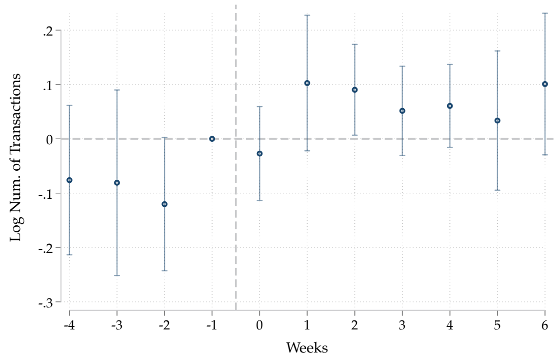
Notes: Panel (a)-(c) MTO data. Author’s calculations. We aggregate daily data into weekly to deal with seasonality. Panel (d) Bloomberg daily Data.

B.5.2 India

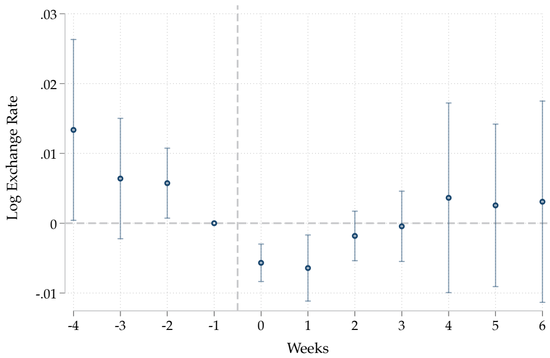
Figure B.11: Event Study: Typhoon "Yaas" in India, 25/05/2021.



(a) Log Amount Sent USD.



(b) Log Total Transactions.

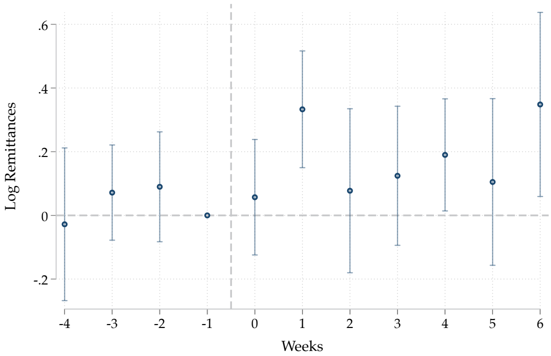


(c) Log Exchange Rate.

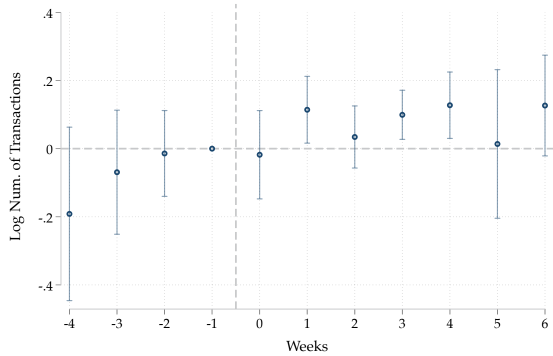
Notes: Panel (a)-(c) MTO data. Author's calculations. We aggregate daily data into weekly to deal with seasonality. Panel (b) Bloomberg daily Data.

B.5.3 Bangladesh

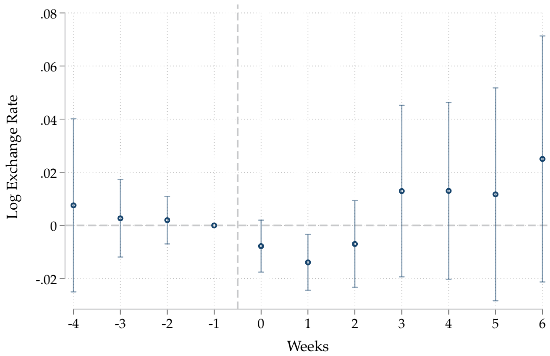
Figure B.12: Event Study: Typhoon "Sitrang" in Bangladesh, 24/10/2022.



(a) Log Amount Sent in USD.



(b) Log Total Transactions.



(c) Log Exchange Rate.

Notes: Panel (a)-(c) MTO data. Author's calculations. We aggregate daily data into weekly to deal with seasonality. Panel (b) Bloomberg daily Data.

B.6 COVID-19

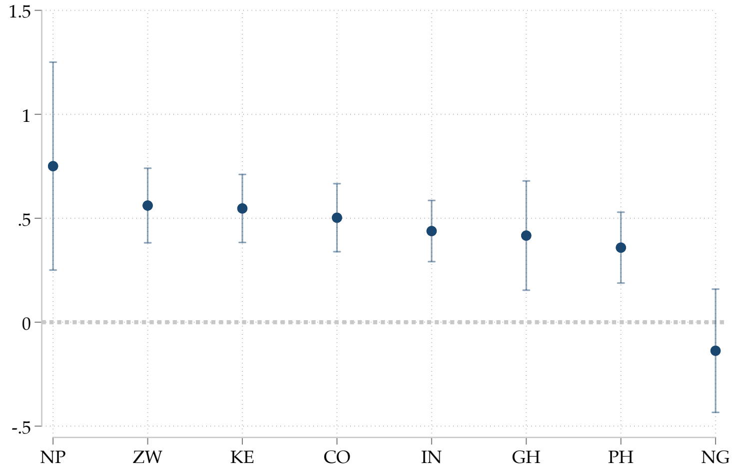
B.6.1 Easing of Lockdown Restrictions

Table B.4: Estimated cumulative Average Treatment on Treated (ATT) by Lockdown-Easing Group

	log Remittances
ATT	0.43***
	(0.10)
May 2020	0.46***
	(0.16)
June 2020	0.38***
	(0.09)

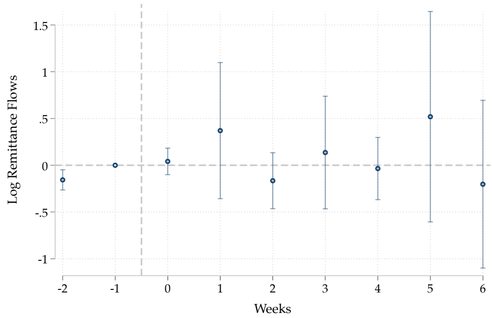
Standard errors in parentheses
* p-value< 0.10, ** p-value< 0.05, *** p-value< 0.01

Figure B.13: Coefficients and Confidence Intervals of the Average Treatment Effect by country

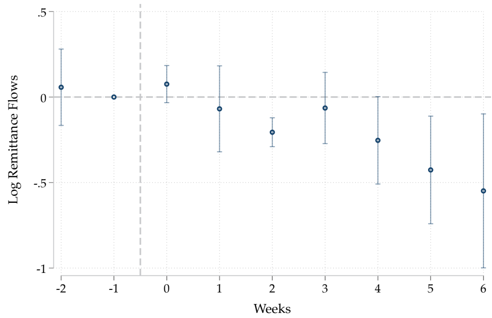


B.6.2 The Stimulus Check

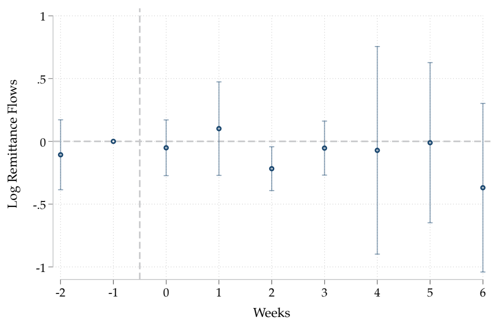
Figure B.14: (Log) Remittances in USD.



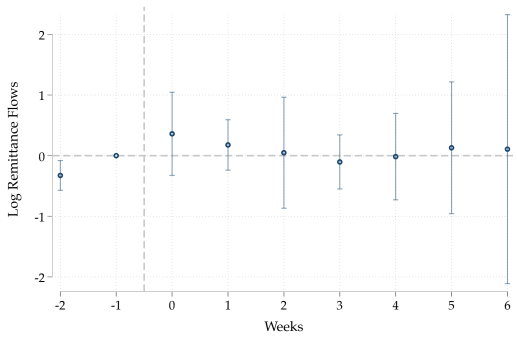
(a) Colombia



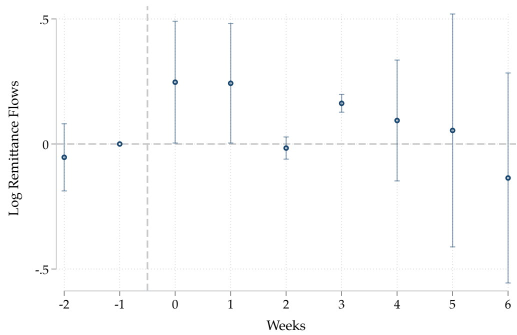
(b) Ghana



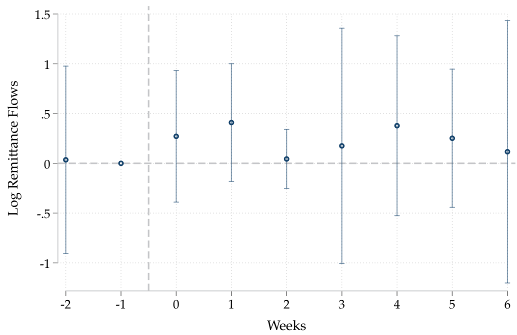
(c) India



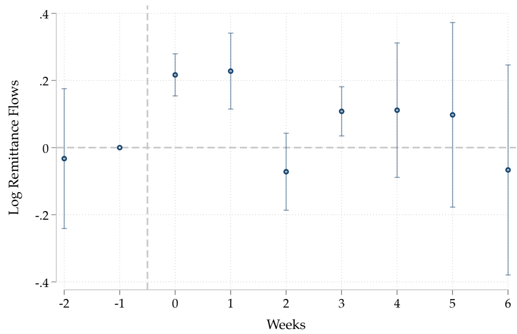
(d) Mexico



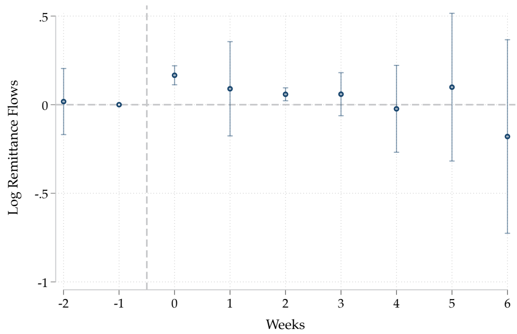
(e) Nigeria



(f) Nepal



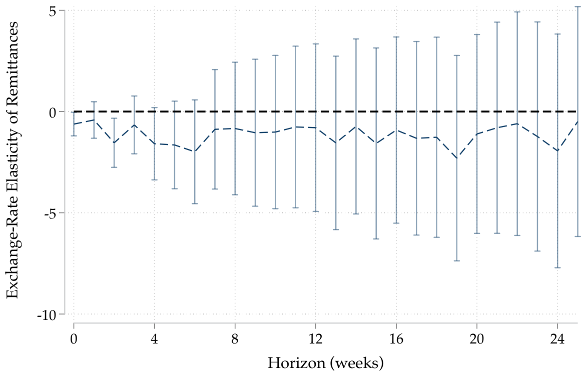
(g) Philippines



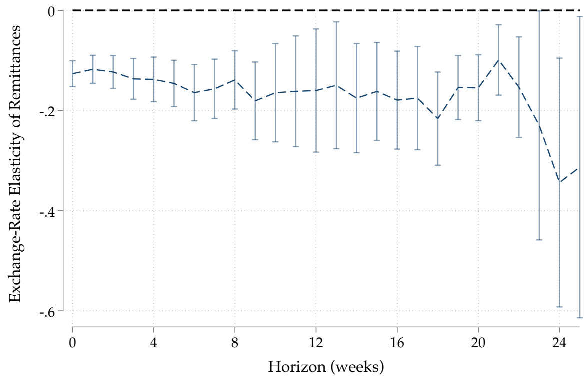
(h) Zimbabwe

B.7 Exchange-Rate Elasticity of Remittances

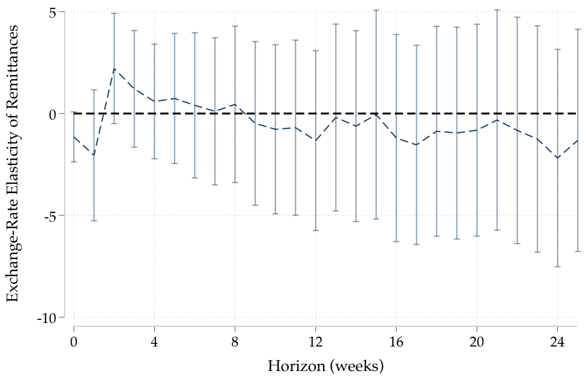
Figure B.15: Exchange-Rate Elasticity of Remittances By Corridors



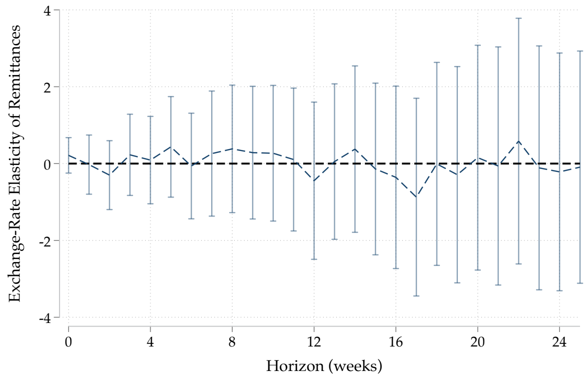
(a) U.S. – Philippines - Post 2020



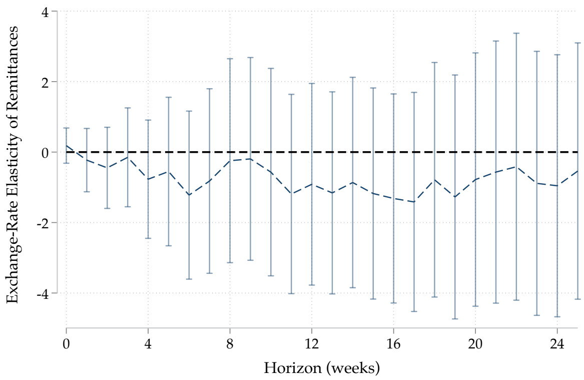
(b) U.S. – Nigeria - Post 2020



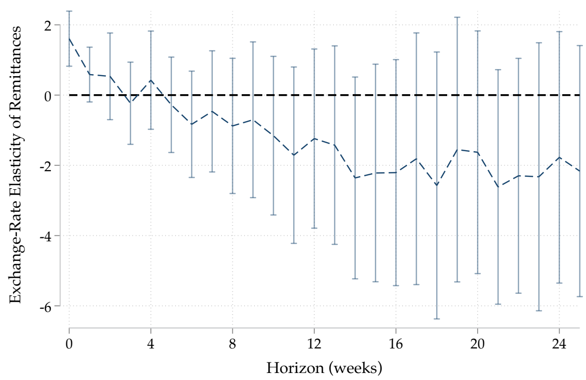
(c) United Kingdom – Zimbabwe - Post 2019



(d) Australia – Philippines - Post 2019



(e) Canada – Philippines - Post 2019



(f) Australia – India - Post 2019

C Model Appendix

C.1 Proofs

This section details the proofs of Propositions 1, 2 and 3, and Corollaries 1 and 2.

C.1.1 Proof of Proposition 1

Proof. Define the pre-remittance disposable income sequences

$$Z_s \equiv \frac{P_{Hs}}{P_s}(Y_s - T_s), \qquad Z_s^* \equiv \frac{P_{Fs}^*}{P_s^*}(Y_s^* - T_s^*).$$

Inclusive of remittance flows, disposable incomes are

$$\widetilde{Z}_s = Z_s + Q_s \omega_s^*, \qquad \widetilde{Z}_s^* = Z_s^* - \omega_s^*.$$

Totally differentiating around the steady state gives

$$d\widetilde{\mathbf{Z}} = d\mathbf{Z} + \overline{\omega}^* d\mathbf{Q} + \overline{Q} d\boldsymbol{\omega}^*, \qquad d\widetilde{\mathbf{Z}}^* = d\mathbf{Z}^* - d\boldsymbol{\omega}^*.$$

Let

$$M_{t,s} \equiv \frac{\partial \mathcal{C}_t}{\partial \widetilde{Z}_s}, \qquad M_{t,s}^* \equiv \frac{\partial \mathcal{C}_t^*}{\partial \widetilde{Z}_s^*}$$

denote the intertemporal marginal propensities to consume out of remittance-inclusive disposable income. Stacking over time,

$$d\mathbf{C} = \mathbf{M} d\widetilde{\mathbf{Z}}, \qquad d\mathbf{C}^* = \mathbf{M}^* d\widetilde{\mathbf{Z}}^*.$$

The remittance policy function satisfies

$$d\boldsymbol{\omega}^* = \boldsymbol{\Omega}_{\mathbf{Z}}^* d\mathbf{Z} + \boldsymbol{\Omega}_{\mathbf{Z}^*}^* d\mathbf{Z}^*,$$

where

$$\Omega_{Z,t,s}^* \equiv \frac{\partial \Omega_t^*}{\partial Z_s}, \qquad \Omega_{Z^*,t,s}^* \equiv \frac{\partial \Omega_t^*}{\partial Z_s^*}.$$

Substituting this expression into the differentiated consumption functions yields

$$d\mathbf{C} = \mathbf{M}\left(\mathbf{I} + \overline{Q}\boldsymbol{\Omega}_{\mathbf{Z}}^*\right) d\mathbf{Z} + \overline{\omega}^* \mathbf{M} d\mathbf{Q} + \overline{Q}\mathbf{M}\boldsymbol{\Omega}_{\mathbf{Z}^*}^* d\mathbf{Z}^*,$$

and

$$d\mathbf{C}^* = \mathbf{M}^* \left(\mathbf{I} - \boldsymbol{\Omega}_{\mathbf{Z}^*}^*\right) d\mathbf{Z}^* - \mathbf{M}^* \boldsymbol{\Omega}_{\mathbf{Z}}^* d\mathbf{Z}.$$

Under the assumptions used in the proposition, the real exchange rate is unchanged, so

$$d\mathbf{Q} = \mathbf{0}.$$

Hence, the term involving $\overline{\omega}^*$ vanishes even if steady-state remittances are nonzero. The previous expressions simplify to

$$d\mathbf{C} = \mathbf{M}\left(\mathbf{I} + \overline{Q}\boldsymbol{\Omega}_Z^*\right) d\mathbf{Z} + \overline{Q}\mathbf{M}\boldsymbol{\Omega}_{Z^*}^* d\mathbf{Z}^*,$$

and

$$d\mathbf{C}^* = \mathbf{M}^* \left(\mathbf{I} - \boldsymbol{\Omega}_{Z^*}^*\right) d\mathbf{Z}^* - \mathbf{M}^* \boldsymbol{\Omega}_Z^* d\mathbf{Z}.$$

These are the consumption impulse equations in Proposition 1. It remains to derive the remittance impulse. The family risk-sharing condition implies

$$d\mathbf{C} = \kappa \, d\mathbf{C}^*.$$

Before substituting the derivative of the remittance policy function, the consumption responses are

$$d\mathbf{C} = \mathbf{M}\left(d\mathbf{Z} + \overline{Q}\, d\boldsymbol{\omega}^*\right), \qquad d\mathbf{C}^* = \mathbf{M}^*\left(d\mathbf{Z}^* - d\boldsymbol{\omega}^*\right).$$

Imposing $d\mathbf{C} = \kappa d\mathbf{C}^*$ gives

$$\mathbf{M}d\mathbf{Z} + \overline{Q}\mathbf{M}d\boldsymbol{\omega}^* = \kappa\mathbf{M}^*d\mathbf{Z}^* - \kappa\mathbf{M}^*d\boldsymbol{\omega}^*.$$

Collecting terms in $d\boldsymbol{\omega}^*$,

$$\left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right) d\boldsymbol{\omega}^* = \kappa\mathbf{M}^*d\mathbf{Z}^* - \mathbf{M}d\mathbf{Z}.$$

This is the remittance equation in Proposition 1. Substituting

$$d\boldsymbol{\omega}^* = \boldsymbol{\Omega}^*_Z d\mathbf{Z} + \boldsymbol{\Omega}^*_{Z^*} d\mathbf{Z}^*$$

and equating coefficients on the arbitrary sequences $d\mathbf{Z}$ and $d\mathbf{Z}^*$ gives

$$\left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right)\boldsymbol{\Omega}^*_Z = -\mathbf{M}, \qquad \left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right)\boldsymbol{\Omega}^*_{Z^*} = \kappa\mathbf{M}^*.$$

These are the iMPR restrictions in Proposition 1. □

Proof of Corollaries 1 and 2. Under the condition in Corollary 1, $\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*$ is boundedly invertible on χ. Applying its inverse to the remittance equation gives

$$d\boldsymbol{\omega}^* = \left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right)^{-1}\left(\kappa\mathbf{M}^*d\mathbf{Z}^* - \mathbf{M}d\mathbf{Z}\right).$$

This proves the unique bounded solution for the remittance impulse in Corollary 1. Comparing coefficients on $d\mathbf{Z}$ and $d\mathbf{Z}^*$ gives

$$\boldsymbol{\Omega}^*_Z = -\left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right)^{-1}\mathbf{M}, \qquad \boldsymbol{\Omega}^*_{Z^*} = \left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right)^{-1}\kappa\mathbf{M}^*.$$

These are the closed-form iMPR representations in Corollary 1. Finally, suppose that Home and Foreign families are symmetric, as in Corollary 2. Then

$$\overline{Q} = 1, \qquad \kappa = 1, \qquad \mathbf{M} = \mathbf{M}^*.$$

Therefore,

$$d\boldsymbol{\omega}^* = (2\mathbf{M})^{-1}\mathbf{M}\left(d\mathbf{Z}^* - d\mathbf{Z}\right) = \frac{1}{2}\left(d\mathbf{Z}^* - d\mathbf{Z}\right).$$

Comparing coefficients on $d\mathbf{Z}$ and $d\mathbf{Z}^*$ gives

$$\boldsymbol{\Omega}^*_Z = -\frac{1}{2}\mathbf{I}, \qquad \boldsymbol{\Omega}^*_{Z^*} = \frac{1}{2}\mathbf{I}.$$

These are the remittance impulse and iMPR operators stated in Corollary 2. □

C.1.2 Proof of Proposition 2

Proof. The goods market clearing conditions, stacked over time and differentiated around the steady state, are

$$d\mathbf{Y} = (1-\alpha)d\mathbf{C} + \overline{Q}\alpha^* d\mathbf{C}^* + d\mathbf{G},$$

and

$$d\mathbf{Y}^* = \overline{Q}^{-1}\alpha d\mathbf{C} + (1-\alpha^*)d\mathbf{C}^* + d\mathbf{G}^*.$$

Under the assumptions of Proposition 2, government spending remains at its steady-state level, so $d\mathbf{G} = d\mathbf{G}^* = \mathbf{0}$. From Proposition 1, the consumption responses satisfy

$$d\mathbf{C} = \mathbf{M}\left(\mathbf{I} + \overline{Q}\mathbf{\Omega}_Z^*\right) d\mathbf{Z} + \overline{\omega}^* \mathbf{M} d\mathbf{Q} + \overline{Q}\mathbf{M}\mathbf{\Omega}_{Z^*}^* d\mathbf{Z}^*,$$

and

$$d\mathbf{C}^* = \mathbf{M}^*\left(\mathbf{I} - \mathbf{\Omega}_{Z^*}^*\right) d\mathbf{Z}^* - \mathbf{M}^*\mathbf{\Omega}_Z^* d\mathbf{Z}.$$

Under the assumptions of Proposition 2, $d\mathbf{Q} = \mathbf{0}$. Hence, the term $\overline{\omega}^*\mathbf{M}d\mathbf{Q}$ vanishes even if steady-state remittances are nonzero. With relative prices unchanged, disposable income satisfies

$$d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}, \qquad d\mathbf{Z}^* = d\mathbf{Y}^* - d\mathbf{T}^*.$$

Substituting the consumption responses into Home goods market clearing gives

$$\begin{aligned} d\mathbf{Y} = &\left[(1-\alpha)\mathbf{M}\left(\mathbf{I} + \overline{Q}\mathbf{\Omega}_Z^*\right) - \overline{Q}\alpha^*\mathbf{M}^*\mathbf{\Omega}_Z^*\right] d\mathbf{Z} \\ &+ \left[(1-\alpha)\overline{Q}\mathbf{M}\mathbf{\Omega}_{Z^*}^* + \overline{Q}\alpha^*\mathbf{M}^*\left(\mathbf{I} - \mathbf{\Omega}_{Z^*}^*\right)\right] d\mathbf{Z}^*. \end{aligned}$$

Similarly, substituting into Foreign goods market clearing gives

$$\begin{aligned} d\mathbf{Y}^* = &\left[\overline{Q}^{-1}\alpha\mathbf{M}\left(\mathbf{I} + \overline{Q}\mathbf{\Omega}_Z^*\right) - (1-\alpha^*)\mathbf{M}^*\mathbf{\Omega}_Z^*\right] d\mathbf{Z} \\ &+ \left[\alpha\mathbf{M}\mathbf{\Omega}_{Z^*}^* + (1-\alpha^*)\mathbf{M}^*\left(\mathbf{I} - \mathbf{\Omega}_{Z^*}^*\right)\right] d\mathbf{Z}^*. \end{aligned}$$

Therefore, using $d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}$ and $d\mathbf{Z}^* = d\mathbf{Y}^* - d\mathbf{T}^*$, the equilibrium output responses satisfy

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \mathbf{\Phi} \begin{pmatrix} d\mathbf{Y} - d\mathbf{T} \\ d\mathbf{Y}^* - d\mathbf{T}^* \end{pmatrix}, \qquad \mathbf{\Phi} \equiv \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix},$$

where

$$\begin{aligned} \Phi_{11} &= (1-\alpha)\mathbf{M}\left(\mathbf{I} + \overline{Q}\mathbf{\Omega}_Z^*\right) - \overline{Q}\alpha^*\mathbf{M}^*\mathbf{\Omega}_Z^*, \\ \Phi_{12} &= (1-\alpha)\overline{Q}\mathbf{M}\mathbf{\Omega}_{Z^*}^* + \overline{Q}\alpha^*\mathbf{M}^*\left(\mathbf{I} - \mathbf{\Omega}_{Z^*}^*\right), \\ \Phi_{21} &= \overline{Q}^{-1}\alpha\mathbf{M}\left(\mathbf{I} + \overline{Q}\mathbf{\Omega}_Z^*\right) - (1-\alpha^*)\mathbf{M}^*\mathbf{\Omega}_Z^*, \\ \Phi_{22} &= \alpha\mathbf{M}\mathbf{\Omega}_{Z^*}^* + (1-\alpha^*)\mathbf{M}^*\left(\mathbf{I} - \mathbf{\Omega}_{Z^*}^*\right). \end{aligned}$$

The four operators above coincide with the blocks of $\mathbf{\Phi}$ stated in Proposition 2. Therefore, the equilibrium output responses satisfy the stated Keynesian cross. □

C.1.3 Lemma for Proposition 3

Lemma 1 (World Goods-Market Adding Up). *Let*

$$\mathbf{B} \equiv \begin{pmatrix} 1-\alpha & \overline{Q}\alpha^* \\ \overline{Q}^{-1}\alpha & 1-\alpha^* \end{pmatrix}$$

denote the fixed-relative-price goods-allocation matrix in local consumption-basket units, and let

$$\mathbf{w}' \equiv \begin{pmatrix} 1 & \overline{Q} \end{pmatrix}$$

denote the common-numeraire world-value weights. Then

$$\mathbf{w}'\mathbf{B} = \mathbf{w}'.$$

If

$$\begin{pmatrix} b_H \\ b_F \end{pmatrix} \equiv \mathbf{B} \begin{pmatrix} \kappa \\ 1 \end{pmatrix}, \qquad \lambda_W \equiv b_H + \overline{Q} b_F,$$

then

$$\lambda_W = \overline{Q} + \kappa.$$

Proof. Direct multiplication gives

$$\begin{aligned} \mathbf{w}'\mathbf{B} &= \begin{pmatrix} 1-\alpha+\overline{Q}\left(\overline{Q}^{-1}\alpha\right) & \overline{Q}\alpha^* + \overline{Q}(1-\alpha^*) \end{pmatrix} \\ &= \begin{pmatrix} 1 & \overline{Q} \end{pmatrix} = \mathbf{w}'. \end{aligned}$$

Therefore,

$$\begin{aligned} \lambda_W &= \mathbf{w}' \begin{pmatrix} b_H \\ b_F \end{pmatrix} \\ &= \mathbf{w}'\mathbf{B} \begin{pmatrix} \kappa \\ 1 \end{pmatrix} \\ &= \mathbf{w}' \begin{pmatrix} \kappa \\ 1 \end{pmatrix} \\ &= \overline{Q} + \kappa. \end{aligned}$$

□

Intuitively, the vector $\mathbf{w}'$ converts Home and Foreign quantities into a common numeraire, while $\mathbf{B}$ allocates Home and Foreign consumption expenditure across the two goods markets. The identity $\mathbf{w}'\mathbf{B} = \mathbf{w}'$ states that this allocation can change where goods are produced without changing the value of aggregate world resources. Risk sharing implies that a one-unit increase in Foreign consumption is accompanied by a κ-unit increase in Home consumption, so the value of the resulting increase in world consumption is $\overline{Q} + \kappa$. The goods-allocation matrix changes the country composition of the associated output response but, by world resource accounting, cannot change its total value. Therefore, $\lambda_W = \overline{Q} + \kappa$. This equality does not require a symmetric steady state. Rather, it is an accounting property that the goods-market coefficients must satisfy when an asymmetric steady state is expressed in a common numeraire.

C.1.4 Proof of Proposition 3

Proof. Define the remittance-inclusive world income sequence

$$d\mathbf{S} \equiv d\mathbf{Z} + \overline{Q} d\mathbf{Z}^*.$$

Under the bounded-invertibility condition in Proposition 3, Corollary 1 implies

$$d\mathbf{C} = \kappa \mathbf{M} \mathbf{A}^{-1} \mathbf{M}^* d\mathbf{S}, \qquad d\mathbf{C}^* = \mathbf{M}^* \mathbf{A}^{-1} \mathbf{M} d\mathbf{S}.$$

The parallel-sum identity implies

$$\mathbf{M} \mathbf{A}^{-1} \mathbf{M}^* = \mathbf{M}^* \mathbf{A}^{-1} \mathbf{M} = \mathbf{H}.$$

Therefore,

$$d\mathbf{C} = \kappa \mathbf{H} d\mathbf{S}, \qquad d\mathbf{C}^* = \mathbf{H} d\mathbf{S}.$$

As

$$d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}, \qquad d\mathbf{Z}^* = d\mathbf{Y}^* - d\mathbf{T}^*,$$

the definitions of world output and world taxes imply

$$d\mathbf{S} = d\mathbf{X} - d\boldsymbol{\tau}^W.$$

Substituting the consumption responses into the Home goods-market equation gives

$$\begin{aligned} d\mathbf{Y} &= \left[(1-\alpha)\kappa + \overline{Q}\alpha^*\right] \mathbf{H} \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right) \\ &= b_H \mathbf{H} \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right). \end{aligned}$$

Similarly, the Foreign goods-market equation gives

$$\begin{aligned} d\mathbf{Y}^* &= \left[\overline{Q}^{-1}\alpha\kappa + (1-\alpha^*)\right] \mathbf{H} \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right) \\ &= b_F \mathbf{H} \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right). \end{aligned}$$

Equivalently, the operator in Proposition 2 can be written as

$$\boldsymbol{\Phi} = \begin{pmatrix} b_H \mathbf{H} & b_H \overline{Q} \mathbf{H} \\ b_F \mathbf{H} & b_F \overline{Q} \mathbf{H} \end{pmatrix}.$$

Multiplying the Home output equation by one, multiplying the Foreign output equation by $\overline{Q}$, and adding gives

$$d\mathbf{X} = \left(b_H + \overline{Q} b_F\right) \mathbf{H} \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right).$$

By the definitions of λ_W and $\mathbf{M}^W$,

$$d\mathbf{X} = \mathbf{M}^W \left(d\mathbf{X} - d\boldsymbol{\tau}^W\right),$$

or, equivalently,

$$\left(\mathbf{I} - \mathbf{M}^W\right) d\mathbf{X} = -\mathbf{M}^W d\boldsymbol{\tau}^W.$$

Multiplying the Home output equation by b_F, multiplying the Foreign output equation by b_H, and taking the difference gives

$$b_F d\mathbf{Y} - b_H d\mathbf{Y}^* = \mathbf{0}.$$

This establishes the world and relative components of the international Keynesian cross. The identity

$$\lambda_W = \overline{Q} + \kappa$$

is the common-numeraire goods-market adding-up identity. To see this, risk sharing implies that world consumption, measured in Home units, is

$$d\mathbf{C} + \overline{Q} d\mathbf{C}^* = \left(\kappa + \overline{Q}\right) \mathbf{H} d\mathbf{S}.$$

The country goods-market equations imply that world output in the same units is

$$\begin{aligned} d\mathbf{Y} + \overline{Q} d\mathbf{Y}^* &= \left(b_H + \overline{Q} b_F\right) \mathbf{H} d\mathbf{S} \\ &= \lambda_W \mathbf{H} d\mathbf{S}. \end{aligned}$$

Thus, world output equals world consumption for every admissible income perturbation precisely when

$$\lambda_W = \overline{Q} + \kappa.$$

Using the definitions of b_H and b_F, the adding-up identity follows directly:

$$\begin{aligned} \lambda_W &= \left[(1-\alpha)\kappa + \overline{Q}\alpha^*\right] + \overline{Q}\left[\overline{Q}^{-1}\alpha\kappa + (1-\alpha^*)\right] \\ &= \kappa + \overline{Q}. \end{aligned}$$

We next establish the present-value adding-up property of the effective world iMPC. Since

$$\mathbf{p}'\mathbf{M} = \mathbf{p}', \qquad \mathbf{p}'\mathbf{M}^* = \mathbf{p}',$$

we have

$$\begin{aligned} \mathbf{p}'\mathbf{A} &= \mathbf{p}'\left(\overline{Q}\mathbf{M} + \kappa\mathbf{M}^*\right) \\ &= \left(\overline{Q} + \kappa\right)\mathbf{p}'. \end{aligned}$$

Bounded invertibility of $\mathbf{A}$ therefore implies

$$\mathbf{p}'\mathbf{A}^{-1} = \frac{1}{\overline{Q} + \kappa}\mathbf{p}'.$$

It follows that

$$\begin{aligned} \mathbf{p}'\mathbf{H} &= \mathbf{p}'\mathbf{M}^*\mathbf{A}^{-1}\mathbf{M} \\ &= \mathbf{p}'\mathbf{A}^{-1}\mathbf{M} \\ &= \frac{1}{\overline{Q} + \kappa}\mathbf{p}'. \end{aligned}$$

Using $\mathbf{M}^W = \lambda_W \mathbf{H}$ and the goods-market adding-up identity,

$$\mathbf{p}'\mathbf{M}^W = \frac{\lambda_W}{\overline{Q} + \kappa}\mathbf{p}' = \mathbf{p}'.$$

Consequently,
$$\mathbf{p}'\left(\mathbf{I}-\mathbf{M}^W\right)=\mathbf{0},$$
so $\mathbf{I}-\mathbf{M}^W$ is not boundedly invertible. Similarly, the compact representation of $\boldsymbol{\Phi}$ implies
$$\begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}\boldsymbol{\Phi}=\begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}.$$
Hence,
$$\begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}(\mathbf{I}_2-\boldsymbol{\Phi})=\mathbf{0},$$
and $\mathbf{I}_2-\boldsymbol{\Phi}$ is not boundedly invertible. It remains to establish the transformed solution. Fiscal feasibility implies
$$\mathbf{p}'d\boldsymbol{\tau}^W=0.$$
Therefore,
$$\begin{aligned}\mathbf{p}'\left(-\mathbf{M}^W d\boldsymbol{\tau}^W\right)&=-\mathbf{p}'d\boldsymbol{\tau}^W\\&=0.\end{aligned}$$

The demand impulse on the right-hand side of the world Keynesian cross therefore has zero present value. Applying the restricted-inverse result of Auclert, Rognlie, and Straub (2024), if
$$\mathbf{K}\left(\mathbf{I}-\mathbf{M}^W\right)$$
is boundedly invertible, the world Keynesian cross has a unique bounded solution given by
$$d\mathbf{X}=-\left[\mathbf{K}\left(\mathbf{I}-\mathbf{M}^W\right)\right]^{-1}\mathbf{K}\mathbf{M}^W d\boldsymbol{\tau}^W.$$
Finally, the relative-output restriction and the definition
$$d\mathbf{X}=d\mathbf{Y}+\overline{Q}d\mathbf{Y}^*$$
imply
$$d\mathbf{Y}=\frac{b_H}{\lambda_W}d\mathbf{X},\qquad d\mathbf{Y}^*=\frac{b_F}{\lambda_W}d\mathbf{X}.$$

□

Discussion. The identity $\lambda_W=\overline{Q}+\kappa$ reflects the closed-world adding-up property. To see this, the iMPC adding-up conditions imply
$$\mathbf{p}'\mathbf{H}=\frac{1}{\overline{Q}+\kappa}\mathbf{p}',\qquad \mathbf{p}'\mathbf{M}^W=\frac{\lambda_W}{\overline{Q}+\kappa}\mathbf{p}'.$$
Thus, the ratio $\lambda_W/(\overline{Q}+\kappa)$ measures the present-value share of an increase in world disposable income that returns as demand for goods produced within the modeled economy. In the closed two-country economy, all expenditure falls on either Home or Foreign goods. Goods-market adding up therefore implies $\lambda_W=\overline{Q}+\kappa$, so that this share equals one. The effective world iMPC then has a unit left eigenvalue, making $\mathbf{I}-\mathbf{M}^W$ noninvertible and requiring the transformation used in Proposition 3.

For the expenditure shares used here, goods-market adding up follows identically from the CES price-index identities and the law-of-one-price conversion factors. In particular,

$$\lambda_W = \left[(1-\alpha)\kappa + \overline{Q}\alpha^*\right] + \overline{Q}\left[\overline{Q}^{-1}\alpha\kappa + (1-\alpha^*)\right] = \kappa + \overline{Q}.$$

This result does not require a symmetric steady state. Under symmetry, $\overline{Q} = 1$, $\gamma = \gamma^*$, and all steady-state relative prices equal one. The expenditure shares then reduce to $\alpha = \alpha^* = \gamma$.

A small-open-economy version would instead treat foreign income, export demand, and remittance inflows from the rest of the world as exogenous. The domestic Keynesian cross could then be written as

$$\left(\mathbf{I} - \mathbf{M}^D\right) d\mathbf{Y} = -\mathbf{M}^D d\mathbf{T} + d\mathbf{D}^{\text{ext}},$$

where $\mathbf{M}^D$ maps domestic disposable income into demand for domestically produced goods, while $d\mathbf{D}^{\text{ext}}$ collects external injections such as export demand and remittance inflows. Because some marginal domestic expenditure falls on imports or is remitted abroad, $\mathbf{M}^D$ need not satisfy the closed-world adding-up condition. The analogue of $\lambda_W/(\overline{Q}+\kappa)$ would then generally be strictly below one. The adding-up argument would therefore no longer force $\mathbf{I} - \mathbf{M}^D$ to be singular. If 1 is not in the spectrum of $\mathbf{M}^D$—with $\rho(\mathbf{M}^D) < 1$ as a sufficient condition—the domestic Keynesian cross can be solved using the ordinary inverse. Endogenizing the rest of the world would restore global adding up and return the analysis to Proposition 3.

C.2 Propositions for Model with Mixed Households

We now adapt the propositions for the case where countries consist of mixed households (individuals and families). Assume that each country has unit population. A mass $\mu \in [0, 1)$ consists of individuals, and a mass $1 - \mu$ consists of family members. We impose $\mu = \mu^*$, ensuring that each cross-border family contains one Home member and one Foreign member.[23]

Let

$$\mathbf{M}_i, \quad \mathbf{M}_f, \qquad \mathbf{M}_i^*, \quad \mathbf{M}_f^*$$

denote the individual and family-member iMPC operators in Home and Foreign. We assume common per-capita income incidence within each country. Thus, individuals and family members receive the same disposable-income impulse $d\mathbf{Z}$ in Home and the same impulse $d\mathbf{Z}^*$ in Foreign.

Let $d\mathbf{x}^*$ denote remittances per family. Aggregate remittances are

$$d\boldsymbol{\omega}^* = (1-\mu)d\mathbf{x}^*. \tag{70}$$

Define

$$\kappa \equiv \frac{\overline{C}_f}{\overline{C}_f^*} = \left(\frac{\eta\overline{Q}}{1-\eta}\right)^{1/\sigma}. \tag{71}$$

Thus, κ is the ratio of family-member consumption, rather than the ratio of aggregate country consumption. Throughout, we maintain that monetary policy is neutral, and therefore

$$d\mathbf{r} = d\mathbf{r}^* = d\mathbf{Q} = \mathbf{0}.$$

Disposable income is

$$d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}, \qquad d\mathbf{Z}^* = d\mathbf{Y}^* - d\mathbf{T}^*. \tag{72}$$

[23] This restriction can be relaxed in a model with non-symmetric country size.

C.2.1 Proposition 4: Consumption and Remittance Impulses with Mixed Households

Proposition 4 (Consumption and Remittance Impulses with Mixed Households). *Suppose the conditional individual and family consumption policy functions and the family remittance policy function are Fréchet differentiable around the steady state. Define the aggregate iMPC operators, holding remittances fixed, by*

$$\mathbf{M} \equiv \mu \mathbf{M}_i + (1-\mu)\mathbf{M}_f, \qquad \mathbf{M}^* \equiv \mu \mathbf{M}_i^* + (1-\mu)\mathbf{M}_f^*. \tag{73}$$

Define the family risk-sharing operator

$$\mathbf{A}_f \equiv \overline{Q}\mathbf{M}_f + \kappa \mathbf{M}_f^*. \tag{74}$$

Then aggregate remittances satisfy

$$\mathbf{A}_f d\boldsymbol{\omega}^* = (1-\mu)\left(\kappa \mathbf{M}_f^* d\mathbf{Z}^* - \mathbf{M}_f d\mathbf{Z}\right). \tag{75}$$

If the aggregate remittance policy is written as

$$d\boldsymbol{\omega}^* = \boldsymbol{\Omega}_Z^* d\mathbf{Z} + \boldsymbol{\Omega}_{Z^*}^* d\mathbf{Z}^*,$$

then the aggregate iMPR operators satisfy

$$\mathbf{A}_f \boldsymbol{\Omega}_Z^* = -(1-\mu)\mathbf{M}_f, \qquad \mathbf{A}_f \boldsymbol{\Omega}_{Z^*}^* = (1-\mu)\kappa \mathbf{M}_f^*. \tag{76}$$

Aggregate consumption satisfies

$$d\mathbf{C} = \left(\mathbf{M} + \overline{Q}\mathbf{M}_f \boldsymbol{\Omega}_Z^*\right) d\mathbf{Z} + \overline{Q}\mathbf{M}_f \boldsymbol{\Omega}_{Z^*}^* d\mathbf{Z}^*, \tag{77}$$

$$d\mathbf{C}^* = -\mathbf{M}_f^* \boldsymbol{\Omega}_Z^* d\mathbf{Z} + \left(\mathbf{M}^* - \mathbf{M}_f^* \boldsymbol{\Omega}_{Z^*}^*\right) d\mathbf{Z}^*. \tag{78}$$

If $\mathbf{A}_f$ is boundedly invertible, the aggregate iMPR operators have the closed-form representations

$$\boldsymbol{\Omega}_Z^* = -(1-\mu)\mathbf{A}_f^{-1}\mathbf{M}_f, \qquad \boldsymbol{\Omega}_{Z^*}^* = (1-\mu)\mathbf{A}_f^{-1}\kappa \mathbf{M}_f^*. \tag{79}$$

If, in addition,

$$\overline{Q} = \kappa = 1, \qquad \mathbf{M}_f = \mathbf{M}_f^*,$$

then

$$d\boldsymbol{\omega}^* = \frac{1-\mu}{2}\left(d\mathbf{Z}^* - d\mathbf{Z}\right), \tag{80}$$

and

$$\boldsymbol{\Omega}_Z^* = -\frac{1-\mu}{2}\mathbf{I}, \qquad \boldsymbol{\Omega}_{Z^*}^* = \frac{1-\mu}{2}\mathbf{I}. \tag{81}$$

Thus, the per-family marginal propensity to remit remains one-half, while the aggregate marginal propensity to remit is scaled by the family mass $1-\mu$.

Proof. Individuals do not send or receive remittances. Their consumption responses are

$$d\mathbf{C}_i = \mathbf{M}_i d\mathbf{Z}, \qquad d\mathbf{C}_i^* = \mathbf{M}_i^* d\mathbf{Z}^*.$$

The consumption responses of the two family members are

$$d\mathbf{C}_f = \mathbf{M}_f \left(d\mathbf{Z} + \overline{Q} d\mathbf{x}^*\right)$$

and

$$d\mathbf{C}_f^* = \mathbf{M}_f^* \left(d\mathbf{Z}^* - d\mathbf{x}^* \right).$$

Under CRRA utility, the family risk-sharing condition is

$$\frac{1-\eta}{\eta} \frac{u'(C_f^*)}{u'(C_f)} = Q.$$

Because $d\mathbf{Q} = \mathbf{0}$, its level linearization gives

$$d\mathbf{C}_f = \kappa d\mathbf{C}_f^*, \qquad \kappa = \frac{\overline{C}_f}{\overline{C}_f^*}.$$

Substituting the family consumption functions gives

$$\mathbf{M}_f \left(d\mathbf{Z} + \overline{Q} d\mathbf{x}^* \right) = \kappa \mathbf{M}_f^* \left(d\mathbf{Z}^* - d\mathbf{x}^* \right).$$

Expanding,

$$\mathbf{M}_f d\mathbf{Z} + \overline{Q} \mathbf{M}_f d\mathbf{x}^* = \kappa \mathbf{M}_f^* d\mathbf{Z}^* - \kappa \mathbf{M}_f^* d\mathbf{x}^*.$$

Collecting the remittance terms yields

$$\left(\overline{Q} \mathbf{M}_f + \kappa \mathbf{M}_f^* \right) d\mathbf{x}^* = \kappa \mathbf{M}_f^* d\mathbf{Z}^* - \mathbf{M}_f d\mathbf{Z}.$$

Since

$$d\boldsymbol{\omega}^* = (1-\mu) d\mathbf{x}^*,$$

multiplying the preceding equation by $1-\mu$ gives

$$\mathbf{A}_f d\boldsymbol{\omega}^* = (1-\mu) \left(\kappa \mathbf{M}_f^* d\mathbf{Z}^* - \mathbf{M}_f d\mathbf{Z} \right).$$

Substituting

$$d\boldsymbol{\omega}^* = \boldsymbol{\Omega}_Z^* d\mathbf{Z} + \boldsymbol{\Omega}_{Z^*}^* d\mathbf{Z}^*$$

and equating coefficients on the arbitrary sequences $d\mathbf{Z}$ and $d\mathbf{Z}^*$ gives

$$\mathbf{A}_f \boldsymbol{\Omega}_Z^* = -(1-\mu) \mathbf{M}_f$$

and

$$\mathbf{A}_f \boldsymbol{\Omega}_{Z^*}^* = (1-\mu) \kappa \mathbf{M}_f^*.$$

We next aggregate consumption. At Home,

$$\begin{aligned} d\mathbf{C} &= \mu d\mathbf{C}_i + (1-\mu) d\mathbf{C}_f \\ &= \mu \mathbf{M}_i d\mathbf{Z} + (1-\mu) \mathbf{M}_f \left(d\mathbf{Z} + \overline{Q} d\mathbf{x}^* \right) \\ &= \left[\mu \mathbf{M}_i + (1-\mu) \mathbf{M}_f \right] d\mathbf{Z} + \overline{Q} \mathbf{M}_f d\boldsymbol{\omega}^* \\ &= \mathbf{M} d\mathbf{Z} + \overline{Q} \mathbf{M}_f d\boldsymbol{\omega}^*. \end{aligned}$$

Similarly, in Foreign,

$$\begin{aligned} d\mathbf{C}^* &= \mu d\mathbf{C}_i^* + (1-\mu)d\mathbf{C}_f^* \\ &= \mu\mathbf{M}_i^* d\mathbf{Z}^* + (1-\mu)\mathbf{M}_f^*\left(d\mathbf{Z}^* - d\mathbf{x}^*\right) \\ &= \left[\mu\mathbf{M}_i^* + (1-\mu)\mathbf{M}_f^*\right] d\mathbf{Z}^* - \mathbf{M}_f^* d\boldsymbol{\omega}^* \\ &= \mathbf{M}^* d\mathbf{Z}^* - \mathbf{M}_f^* d\boldsymbol{\omega}^*. \end{aligned}$$

Substituting the aggregate remittance policy produces equations (77) and (78).

If $\mathbf{A}_f$ is boundedly invertible, applying its inverse gives equation (79). Under family symmetry,

$$\mathbf{A}_f = 2\mathbf{M}_f,$$

which implies

$$\boldsymbol{\Omega}_Z^* = -\frac{1-\mu}{2}\mathbf{I}, \qquad \boldsymbol{\Omega}_{Z^*}^* = \frac{1-\mu}{2}\mathbf{I}.$$

□

C.2.2 Proposition 5: The Mixed-Household International Keynesian Cross

Proposition 5 (The Mixed-Household International Keynesian Cross). *Suppose the assumptions of Proposition 4 hold. Suppose also that government spending remains at its steady-state level,*

$$d\mathbf{G} = d\mathbf{G}^* = \mathbf{0}.$$

Define the aggregate consumption-response operator

$$\boldsymbol{\Psi}_\mu \equiv \begin{pmatrix} \boldsymbol{\Psi}_{11}^\mu & \boldsymbol{\Psi}_{12}^\mu \\ \boldsymbol{\Psi}_{21}^\mu & \boldsymbol{\Psi}_{22}^\mu \end{pmatrix}, \tag{82}$$

where

$$\boldsymbol{\Psi}_{11}^\mu \equiv \mathbf{M} + \overline{Q}\mathbf{M}_f\boldsymbol{\Omega}_Z^*, \tag{83}$$
$$\boldsymbol{\Psi}_{12}^\mu \equiv \overline{Q}\mathbf{M}_f\boldsymbol{\Omega}_{Z^*}^*, \tag{84}$$
$$\boldsymbol{\Psi}_{21}^\mu \equiv -\mathbf{M}_f^*\boldsymbol{\Omega}_Z^*, \tag{85}$$
$$\boldsymbol{\Psi}_{22}^\mu \equiv \mathbf{M}^* - \mathbf{M}_f^*\boldsymbol{\Omega}_{Z^*}^*. \tag{86}$$

Let

$$\mathbf{B} \equiv \begin{pmatrix} 1-\alpha & \overline{Q}\alpha^* \\ \overline{Q}^{-1}\alpha & 1-\alpha^* \end{pmatrix} \tag{87}$$

denote the fixed-relative-price goods-allocation matrix. Then every bounded equilibrium output response satisfies

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \boldsymbol{\Phi}_\mu \begin{pmatrix} d\mathbf{Y} - d\mathbf{T} \\ d\mathbf{Y}^* - d\mathbf{T}^* \end{pmatrix}, \qquad \textit{where } \boldsymbol{\Phi}_\mu = \mathbf{B}\boldsymbol{\Psi}_\mu, \tag{88}$$

and the four blocks of $\mathbf{\Phi}_\mu$ are

$$\Phi^{\mu}_{11} = (1-\alpha)\left(\mathbf{M} + \overline{Q}\mathbf{M}_f\mathbf{\Omega}^*_Z\right) - \overline{Q}\alpha^*\mathbf{M}^*_f\mathbf{\Omega}^*_Z, \tag{89}$$

$$\Phi^{\mu}_{12} = (1-\alpha)\overline{Q}\mathbf{M}_f\mathbf{\Omega}^*_{Z^*} + \overline{Q}\alpha^*\left(\mathbf{M}^* - \mathbf{M}^*_f\mathbf{\Omega}^*_{Z^*}\right), \tag{90}$$

$$\Phi^{\mu}_{21} = \overline{Q}^{-1}\alpha\left(\mathbf{M} + \overline{Q}\mathbf{M}_f\mathbf{\Omega}^*_Z\right) - (1-\alpha^*)\mathbf{M}^*_f\mathbf{\Omega}^*_Z, \tag{91}$$

$$\Phi^{\mu}_{22} = \alpha\mathbf{M}_f\mathbf{\Omega}^*_{Z^*} + (1-\alpha^*)\left(\mathbf{M}^* - \mathbf{M}^*_f\mathbf{\Omega}^*_{Z^*}\right). \tag{92}$$

Proof. At fixed relative prices, goods-market clearing is

$$d\mathbf{Y} = (1-\alpha)d\mathbf{C} + \overline{Q}\alpha^* d\mathbf{C}^*$$

and

$$d\mathbf{Y}^* = \overline{Q}^{-1}\alpha d\mathbf{C} + (1-\alpha^*)d\mathbf{C}^*.$$

Stacking these equations gives

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \mathbf{B}\begin{pmatrix} d\mathbf{C} \\ d\mathbf{C}^* \end{pmatrix}.$$

From Proposition 4,

$$\begin{pmatrix} d\mathbf{C} \\ d\mathbf{C}^* \end{pmatrix} = \mathbf{\Psi}_\mu\begin{pmatrix} d\mathbf{Z} \\ d\mathbf{Z}^* \end{pmatrix}.$$

Therefore,

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \mathbf{B}\mathbf{\Psi}_\mu\begin{pmatrix} d\mathbf{Z} \\ d\mathbf{Z}^* \end{pmatrix}.$$

Since

$$d\mathbf{Z} = d\mathbf{Y} - d\mathbf{T}, \qquad d\mathbf{Z}^* = d\mathbf{Y}^* - d\mathbf{T}^*,$$

we obtain

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \mathbf{\Phi}_\mu\begin{pmatrix} d\mathbf{Y} - d\mathbf{T} \\ d\mathbf{Y}^* - d\mathbf{T}^* \end{pmatrix},$$

where

$$\mathbf{\Phi}_\mu = \mathbf{B}\mathbf{\Psi}_\mu.$$

Direct block multiplication gives

$$\begin{aligned}
\Phi^{\mu}_{11} &= (1-\alpha)\Psi^{\mu}_{11} + \overline{Q}\alpha^*\Psi^{\mu}_{21}, \\
\Phi^{\mu}_{12} &= (1-\alpha)\Psi^{\mu}_{12} + \overline{Q}\alpha^*\Psi^{\mu}_{22}, \\
\Phi^{\mu}_{21} &= \overline{Q}^{-1}\alpha\Psi^{\mu}_{11} + (1-\alpha^*)\Psi^{\mu}_{21}, \\
\Phi^{\mu}_{22} &= \overline{Q}^{-1}\alpha\Psi^{\mu}_{12} + (1-\alpha^*)\Psi^{\mu}_{22}.
\end{aligned}$$

Substituting the four blocks of $\mathbf{\Psi}_\mu$ gives equations (89)–(92). □

C.2.3 Lemmas for Proposition 6

The next two lemmas separate individual demand from family risk sharing and then map these components into world and relative output. They show why, unlike the family-only case in Proposition 3, the mixed-household system generally cannot be reduced to a single dynamic operator equation for the world-output sequence together with a fixed proportionality relation between country outputs.

To characterize aggregate consumption with mixed households, suppose that $\mathbf{A}_f$ is boundedly invertible and define

$$\mathbf{H} \equiv \mathbf{M}_f^* \mathbf{A}_f^{-1} \mathbf{M}_f = \mathbf{M}_f \mathbf{A}_f^{-1} \mathbf{M}_f^*. \tag{93}$$

Define the mass-weighted individual iMPC operators by

$$\mathbf{M}_\mu^i \equiv \mu \mathbf{M}_i, \qquad \mathbf{M}_\mu^{i*} \equiv \mu \mathbf{M}_i^*. \tag{94}$$

Lemma 2 (Aggregate Consumption with Mixed Households). *Suppose the assumptions of Propositions 4 and 5 hold and $\mathbf{A}_f$ is boundedly invertible on χ. Then the aggregate consumption-response operator satisfies*

$$\mathbf{\Psi}_\mu = \begin{pmatrix} \mathbf{M}_\mu^i & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_\mu^{i*} \end{pmatrix} + (1-\mu) \begin{pmatrix} \kappa \mathbf{I} \\ \mathbf{I} \end{pmatrix} \mathbf{H} \begin{pmatrix} \mathbf{I} & \overline{Q}\mathbf{I} \end{pmatrix}. \tag{95}$$

Proof. The remittance equation for one family is

$$\mathbf{A}_f d\mathbf{x}^* = \kappa \mathbf{M}_f^* d\mathbf{Z}^* - \mathbf{M}_f d\mathbf{Z}.$$

Bounded invertibility of $\mathbf{A}_f$ therefore implies

$$d\mathbf{x}^* = \mathbf{A}_f^{-1} \left(\kappa \mathbf{M}_f^* d\mathbf{Z}^* - \mathbf{M}_f d\mathbf{Z} \right).$$

The Home family member's consumption response is

$$\begin{aligned} d\mathbf{C}_f &= \mathbf{M}_f \left(d\mathbf{Z} + \overline{Q} d\mathbf{x}^* \right) \\ &= \mathbf{M}_f \left(\mathbf{I} - \overline{Q} \mathbf{A}_f^{-1} \mathbf{M}_f \right) d\mathbf{Z} \\ &\quad + \overline{Q} \kappa \mathbf{M}_f \mathbf{A}_f^{-1} \mathbf{M}_f^* d\mathbf{Z}^*. \end{aligned}$$

Since

$$\mathbf{A}_f = \overline{Q} \mathbf{M}_f + \kappa \mathbf{M}_f^*,$$

we have

$$\mathbf{I} - \overline{Q} \mathbf{A}_f^{-1} \mathbf{M}_f = \kappa \mathbf{A}_f^{-1} \mathbf{M}_f^*.$$

Moreover,

$$\begin{aligned} \mathbf{M}_f \mathbf{A}_f^{-1} \mathbf{M}_f^* &= \frac{1}{\kappa} \left(\mathbf{M}_f - \overline{Q} \mathbf{M}_f \mathbf{A}_f^{-1} \mathbf{M}_f \right) \\ &= \mathbf{M}_f^* \mathbf{A}_f^{-1} \mathbf{M}_f = \mathbf{H}. \end{aligned}$$

It follows that

$$d\mathbf{C}_f = \kappa \mathbf{H} \left(d\mathbf{Z} + \overline{Q} d\mathbf{Z}^* \right).$$

Similarly, the Foreign family member's consumption response is

$$\begin{aligned} d\mathbf{C}_f^* &= \mathbf{M}_f^* \left(d\mathbf{Z}^* - d\mathbf{x}^*\right) \\ &= \mathbf{M}_f^* \left(\mathbf{I} - \kappa \mathbf{A}_f^{-1} \mathbf{M}_f^*\right) d\mathbf{Z}^* + \mathbf{M}_f^* \mathbf{A}_f^{-1} \mathbf{M}_f d\mathbf{Z}. \end{aligned}$$

Using

$$\mathbf{I} - \kappa \mathbf{A}_f^{-1} \mathbf{M}_f^* = \overline{Q} \mathbf{A}_f^{-1} \mathbf{M}_f,$$

we obtain

$$d\mathbf{C}_f^* = \mathbf{H}\left(d\mathbf{Z} + \overline{Q} d\mathbf{Z}^*\right).$$

Aggregating individual and family consumption gives

$$d\mathbf{C} = \mathbf{M}_\mu^i d\mathbf{Z} + (1-\mu)\kappa \mathbf{H}\left(d\mathbf{Z} + \overline{Q} d\mathbf{Z}^*\right)$$

and

$$d\mathbf{C}^* = \mathbf{M}_\mu^{i*} d\mathbf{Z}^* + (1-\mu)\mathbf{H}\left(d\mathbf{Z} + \overline{Q} d\mathbf{Z}^*\right).$$

Stacking these equations yields equation (95). □

To state the world and relative-output implications, define

$$d\mathbf{X} \equiv d\mathbf{Y} + \overline{Q} d\mathbf{Y}^*, \qquad d\boldsymbol{\tau}^W \equiv d\mathbf{T} + \overline{Q} d\mathbf{T}^*. \tag{96}$$

Define the goods-market coefficients

$$b_H \equiv (1-\alpha)\kappa + \overline{Q}\alpha^*, \qquad b_F \equiv \overline{Q}^{-1}\alpha\kappa + (1-\alpha^*). \tag{97}$$

Finally, define the country-specific world iMPC operators

$$\mathbf{M}_H^W \equiv \mathbf{M}_\mu^i + (1-\mu)(\kappa + \overline{Q})\mathbf{H}, \qquad \mathbf{M}_F^W \equiv \mathbf{M}_\mu^{i*} + (1-\mu)(\kappa + \overline{Q})\mathbf{H}. \tag{98}$$

Lemma 3 (World and Relative-Output Decomposition). *Under the conditions of Lemma 2, every bounded equilibrium satisfies*

$$d\mathbf{X} = \mathbf{M}_H^W d\mathbf{Z} + \overline{Q}\mathbf{M}_F^W d\mathbf{Z}^* \tag{99}$$

and

$$b_F d\mathbf{Y} - b_H d\mathbf{Y}^* = \det(\mathbf{B})\left(\mathbf{M}_\mu^i d\mathbf{Z} - \kappa \mathbf{M}_\mu^{i*} d\mathbf{Z}^*\right), \tag{100}$$

where $\det(\mathbf{B}) = 1 - \alpha - \alpha^*$. *The world-output equation reduces to*

$$\left(\mathbf{I} - \mathbf{M}_\mu^W\right) d\mathbf{X} = -\mathbf{M}_\mu^W d\boldsymbol{\tau}^W \tag{101}$$

if and only if

$$\mathbf{M}_\mu^i = \mathbf{M}_\mu^{i*}. \tag{102}$$

Under this condition,

$$\mathbf{M}_\mu^W = \mathbf{M}_\mu^i + (1-\mu)(\kappa + \overline{Q})\mathbf{H}. \tag{103}$$

Proof. The goods-allocation matrix satisfies

$$\begin{pmatrix} \mathbf{I} & \overline{Q}\mathbf{I} \end{pmatrix} \mathbf{B} = \begin{pmatrix} \mathbf{I} & \overline{Q}\mathbf{I} \end{pmatrix}.$$

Thus, goods-market clearing implies

$$d\mathbf{X} = d\mathbf{C} + \overline{Q}d\mathbf{C}^*.$$

Using Lemma 2 gives

$$\begin{aligned} d\mathbf{X} = &\left[\mathbf{M}^i_\mu + (1-\mu)(\kappa + \overline{Q})\mathbf{H}\right] d\mathbf{Z} \\ &+ \overline{Q}\left[\mathbf{M}^{i*}_\mu + (1-\mu)(\kappa + \overline{Q})\mathbf{H}\right] d\mathbf{Z}^*. \end{aligned}$$

This proves equation (99). The world-output equation depends only on

$$d\mathbf{Z} + \overline{Q}d\mathbf{Z}^*$$

if and only if

$$\mathbf{M}^W_H = \mathbf{M}^W_F.$$

The family components of these two operators are identical. Therefore,

$$\mathbf{M}^W_H = \mathbf{M}^W_F$$

if and only if

$$\mathbf{M}^i_\mu = \mathbf{M}^{i*}_\mu.$$

Under this condition,

$$d\mathbf{X} = \mathbf{M}^W_\mu \left(d\mathbf{Z} + \overline{Q}d\mathbf{Z}^*\right).$$

Since

$$d\mathbf{Z} + \overline{Q}d\mathbf{Z}^* = d\mathbf{X} - d\boldsymbol{\tau}^W,$$

we obtain equation (101).

It remains to establish the relative-output equation. From Lemma 2 and goods-market clearing,

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix} = \mathbf{B}\left[\begin{pmatrix} \mathbf{M}^i_\mu d\mathbf{Z} \\ \mathbf{M}^{i*}_\mu d\mathbf{Z}^* \end{pmatrix} + (1-\mu)\begin{pmatrix} \kappa\mathbf{I} \\ \mathbf{I} \end{pmatrix}\mathbf{H}\left(d\mathbf{Z} + \overline{Q}d\mathbf{Z}^*\right)\right].$$

By the definitions of b_H and b_F,

$$\mathbf{B}\begin{pmatrix} \kappa\mathbf{I} \\ \mathbf{I} \end{pmatrix} = \begin{pmatrix} b_H\mathbf{I} \\ b_F\mathbf{I} \end{pmatrix}.$$

The family component therefore cancels from

$$b_F d\mathbf{Y} - b_H d\mathbf{Y}^*.$$

The remaining individual component gives

$$\begin{aligned} b_F d\mathbf{Y} - b_H d\mathbf{Y}^* = &\left[b_F(1-\alpha) - b_H\overline{Q}^{-1}\alpha\right]\mathbf{M}^i_\mu d\mathbf{Z} \\ &+ \left[b_F\overline{Q}\alpha^* - b_H(1-\alpha^*)\right]\mathbf{M}^{i*}_\mu d\mathbf{Z}^*. \end{aligned}$$

Direct substitution gives

$$b_F(1-\alpha) - b_H\overline{Q}^{-1}\alpha = 1 - \alpha - \alpha^*$$

and

$$b_F\overline{Q}\alpha^* - b_H(1-\alpha^*) = -\kappa(1-\alpha-\alpha^*).$$

This proves equation (100). □

Intuitively, the determinant appears because the relative-output combination removes the family-generated component of demand. The remaining individual-demand component is scaled by $\det(\mathbf{B})$, which measures whether the goods-allocation matrix preserves differences between Home and Foreign demand. When $\det(\mathbf{B}) = 0$, the allocation matrix has rank one, and country-specific demand changes generate the same composition of output across countries.

C.2.4 Proposition 6: Solving the Mixed-Household International Keynesian Cross

Proposition 6 (Solving the Mixed-Household International Keynesian Cross). *Suppose the conditions of Lemmas 2 and 3 hold and $\bar{r} > 0$. Let $\mathbf{p}$ denote the present-value weights, with $p_t = (1+\bar{r})^{-t}$. Suppose*

$$\mathbf{p}'\mathbf{M}_i = \mathbf{p}', \qquad \mathbf{p}'\mathbf{M}_i^* = \mathbf{p}', \qquad \mathbf{p}'\mathbf{M}_f = \mathbf{p}', \qquad \mathbf{p}'\mathbf{M}_f^* = \mathbf{p}'. \tag{104}$$

Then

$$\begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}\boldsymbol{\Phi}_\mu = \begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}. \tag{105}$$

Consequently, $\mathbf{I}_2 - \boldsymbol{\Phi}_\mu$ is not boundedly invertible. Define

$$\mathbf{y} \equiv \begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^* \end{pmatrix}, \qquad \mathbf{t} \equiv \begin{pmatrix} d\mathbf{T} \\ d\mathbf{T}^* \end{pmatrix},$$

$$\mathcal{T} \equiv \begin{pmatrix} \mathbf{I} & \overline{Q}\mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{pmatrix}, \tag{106}$$

and

$$d\mathbf{Y}^\Delta \equiv d\mathbf{Y} - d\mathbf{Y}^*, \qquad d\mathbf{T}^\Delta \equiv d\mathbf{T} - d\mathbf{T}^*. \tag{107}$$

Let

$$\tilde{\mathbf{y}} \equiv \mathcal{T}\mathbf{y} = \begin{pmatrix} d\mathbf{X} \\ d\mathbf{Y}^\Delta \end{pmatrix}, \qquad \tilde{\mathbf{t}} \equiv \mathcal{T}\mathbf{t} = \begin{pmatrix} d\boldsymbol{\tau}^W \\ d\mathbf{T}^\Delta \end{pmatrix}. \tag{108}$$

Define

$$\widetilde{\boldsymbol{\Phi}}_\mu \equiv \mathcal{T}\boldsymbol{\Phi}_\mu\mathcal{T}^{-1}. \tag{109}$$

Let $\mathbf{F}$ denote the lead operator and define, as in Proposition 3,

$$\mathbf{K} \equiv -\sum_{j=1}^{\infty}(1+\bar{r})^{-j}\mathbf{F}^j, \qquad \mathcal{K} \equiv \begin{pmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}. \tag{110}$$

For any fiscally feasible tax shock satisfying

$$\mathbf{p}'d\boldsymbol{\tau}^W = 0, \tag{111}$$

if

$$\mathcal{K}\left(\mathbf{I}_2 - \widetilde{\boldsymbol{\Phi}}_\mu\right) \tag{112}$$

is boundedly invertible, then the mixed-household international Keynesian cross has a unique bounded solution given by

$$\widetilde{\mathbf{y}} = -\left[\mathcal{K}\left(\mathbf{I}_2 - \widetilde{\mathbf{\Phi}}_\mu\right)\right]^{-1}\mathcal{K}\widetilde{\mathbf{\Phi}}_\mu\widetilde{\mathbf{t}}, \qquad \mathbf{y} = \mathcal{T}^{-1}\widetilde{\mathbf{y}}. \tag{113}$$

Proof. The family iMPC adding-up conditions imply

$$\mathbf{p}'\mathbf{A}_f = \mathbf{p}'\left(\overline{Q}\mathbf{M}_f + \kappa\mathbf{M}_f^*\right) = (\overline{Q}+\kappa)\mathbf{p}'.$$

Since $\mathbf{A}_f$ is boundedly invertible,

$$\mathbf{p}'\mathbf{A}_f^{-1} = \frac{1}{\overline{Q}+\kappa}\mathbf{p}'.$$

It follows that

$$\begin{aligned}\mathbf{p}'\mathbf{H} &= \mathbf{p}'\mathbf{M}_f^*\mathbf{A}_f^{-1}\mathbf{M}_f\\ &= \mathbf{p}'\mathbf{A}_f^{-1}\mathbf{M}_f\\ &= \frac{1}{\overline{Q}+\kappa}\mathbf{p}'.\end{aligned}$$

The individual iMPC adding-up conditions imply

$$\mathbf{p}'\mathbf{M}_\mu^i = \mu\mathbf{p}', \qquad \mathbf{p}'\mathbf{M}_\mu^{i*} = \mu\mathbf{p}'.$$

Using Lemma 2,

$$\begin{aligned}&\left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right)\mathbf{\Psi}_\mu\\ &= \mu\left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right)\\ &\quad + (1-\mu)(\kappa+\overline{Q})\mathbf{p}'\mathbf{H}\left(\mathbf{I} \quad \overline{Q}\mathbf{I}\right)\\ &= \left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right).\end{aligned}$$

The goods-allocation identity gives

$$\left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right)\mathbf{B} = \left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right).$$

Since

$$\mathbf{\Phi}_\mu = \mathbf{B}\mathbf{\Psi}_\mu,$$

equation (105) follows. Therefore,

$$\left(\mathbf{p}' \quad \overline{Q}\mathbf{p}'\right)\left(\mathbf{I}_2 - \mathbf{\Phi}_\mu\right) = \mathbf{0}.$$

Thus, $\mathbf{I}_2 - \mathbf{\Phi}_\mu$ has a nonzero left null vector and is not boundedly invertible.

The Keynesian cross in Proposition 5 can be written as

$$\left(\mathbf{I}_2 - \mathbf{\Phi}_\mu\right)\mathbf{y} = -\mathbf{\Phi}_\mu\mathbf{t}.$$

Premultiplying by $\mathcal{T}$ and using

$$\widetilde{\mathbf{\Phi}}_\mu = \mathcal{T}\mathbf{\Phi}_\mu\mathcal{T}^{-1}$$

gives

$$\left(\mathbf{I}_2 - \widetilde{\mathbf{\Phi}}_\mu\right)\widetilde{\mathbf{y}} = -\widetilde{\mathbf{\Phi}}_\mu\widetilde{\mathbf{t}}.$$

The left null vector in the transformed coordinates is

$$\begin{pmatrix}\mathbf{p}' & \overline{Q}\mathbf{p}'\end{pmatrix}\mathcal{T}^{-1} = \begin{pmatrix}\mathbf{p}' & \mathbf{0}\end{pmatrix}.$$

Thus, the singular direction lies entirely in the world-output equation. Moreover,

$$\begin{aligned}\begin{pmatrix}\mathbf{p}' & \mathbf{0}\end{pmatrix}\left(-\widetilde{\mathbf{\Phi}}_\mu\widetilde{\mathbf{t}}\right) &= -\begin{pmatrix}\mathbf{p}' & \mathbf{0}\end{pmatrix}\widetilde{\mathbf{t}} \\ &= -\mathbf{p}'d\boldsymbol{\tau}^W \\ &= 0,\end{aligned}$$

where the final equality follows from fiscal feasibility. Pre-multiplication by

$$\mathcal{K} = \mathrm{diag}(\mathbf{K}, \mathbf{I})$$

applies the restricted-inverse transformation from Proposition 3 to the world-output equation while leaving the relative-output equation unchanged. Under the stated bounded-invertibility condition, the transformed system therefore has the unique bounded solution

$$\widetilde{\mathbf{y}} = -\left[\mathcal{K}\left(\mathbf{I}_2 - \widetilde{\mathbf{\Phi}}_\mu\right)\right]^{-1}\mathcal{K}\widetilde{\mathbf{\Phi}}_\mu\widetilde{\mathbf{t}}.$$

Finally,

$$\mathbf{y} = \mathcal{T}^{-1}\widetilde{\mathbf{y}},$$

which proves the result. □

Corollary 3 (Symmetric Mixed Households). *Suppose the conditions of Proposition 6 hold and, in addition,*

$$\overline{Q} = \kappa = 1, \qquad \alpha = \alpha^*, \qquad \mathbf{M}_i = \mathbf{M}_i^*, \qquad \mathbf{M}_f = \mathbf{M}_f^*. \tag{114}$$

Then

$$\mathbf{H} = \frac{1}{2}\mathbf{M}_f.$$

Define

$$\mathbf{M}_\mu^W \equiv \mu\mathbf{M}_i + (1-\mu)\mathbf{M}_f. \tag{115}$$

The world and relative-output systems decouple:

$$\left(\mathbf{I} - \mathbf{M}_\mu^W\right)d\mathbf{X} = -\mathbf{M}_\mu^W d\boldsymbol{\tau}^W \tag{116}$$

and

$$\left[\mathbf{I} - (1-2\alpha)\mu\mathbf{M}_i\right]d\mathbf{Y}^\Delta = -(1-2\alpha)\mu\mathbf{M}_i d\mathbf{T}^\Delta. \tag{117}$$

If

$$\mathbf{K}\left(\mathbf{I} - \mathbf{M}_\mu^W\right)$$

and

$$\mathbf{I} - (1-2\alpha)\mu\mathbf{M}_i$$

are boundedly invertible, the solutions are

$$d\mathbf{X} = -\left[\mathbf{K}\left(\mathbf{I} - \mathbf{M}_{\mu}^{W}\right)\right]^{-1}\mathbf{K}\mathbf{M}_{\mu}^{W} d\boldsymbol{\tau}^{W} \tag{118}$$

and

$$d\mathbf{Y}^{\Delta} = -\left[\mathbf{I} - (1-2\alpha)\mu\mathbf{M}_i\right]^{-1}(1-2\alpha)\mu\mathbf{M}_i d\mathbf{T}^{\Delta}. \tag{119}$$

Country outputs are recovered from

$$d\mathbf{Y} = \frac{1}{2}\left(d\mathbf{X} + d\mathbf{Y}^{\Delta}\right), \qquad d\mathbf{Y}^{*} = \frac{1}{2}\left(d\mathbf{X} - d\mathbf{Y}^{\Delta}\right). \tag{120}$$

At $\mu = 0$, the relative-output response is zero and Proposition 3 is recovered.

Proof. Under symmetry,

$$\mathbf{A}_f = 2\mathbf{M}_f.$$

Therefore,

$$\mathbf{H} = \mathbf{M}_f\left(2\mathbf{M}_f\right)^{-1}\mathbf{M}_f = \frac{1}{2}\mathbf{M}_f.$$

Lemma 2 then implies

$$\boldsymbol{\Psi}_{\mu} = \begin{pmatrix} \mu\mathbf{M}_i & \mathbf{0} \\ \mathbf{0} & \mu\mathbf{M}_i \end{pmatrix} + \frac{1-\mu}{2}\begin{pmatrix} \mathbf{I} \\ \mathbf{I} \end{pmatrix}\mathbf{M}_f\begin{pmatrix} \mathbf{I} & \mathbf{I} \end{pmatrix}.$$

Under symmetry, the goods-allocation matrix is

$$\mathbf{B} = \begin{pmatrix} (1-\alpha)\mathbf{I} & \alpha\mathbf{I} \\ \alpha\mathbf{I} & (1-\alpha)\mathbf{I} \end{pmatrix}.$$

The world direction is an eigenvector of $\mathbf{B}$ with eigenvalue one, while the relative direction is an eigenvector with eigenvalue $1-2\alpha$. Family demand appears only in the world direction. Individual demand appears in both directions. Consequently,

$$\widetilde{\boldsymbol{\Phi}}_{\mu} = \begin{pmatrix} \mathbf{M}_{\mu}^{W} & \mathbf{0} \\ \mathbf{0} & (1-2\alpha)\mu\mathbf{M}_i \end{pmatrix}.$$

Substituting this expression into the transformed Keynesian cross gives equations (116) and (117). Applying the restricted inverse to the world equation and the ordinary inverse to the relative equation gives equations (118) and (119).

Finally, when $\overline{Q} = 1$,

$$\mathcal{T}^{-1} = \frac{1}{2}\begin{pmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{pmatrix}.$$

Therefore,

$$\begin{pmatrix} d\mathbf{Y} \\ d\mathbf{Y}^{*} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{I} & -\mathbf{I} \end{pmatrix}\begin{pmatrix} d\mathbf{X} \\ d\mathbf{Y}^{\Delta} \end{pmatrix},$$

which proves equation (120). At $\mu = 0$, equation (117) implies $d\mathbf{Y}^{\Delta} = \mathbf{0}$, while $\mathbf{M}_{\mu}^{W} = \mathbf{M}_f$. Hence, the system reduces to Proposition 3. $\square$

C.3 Perpetual Youth OLG Model with Remittance Flows

In this section, we further detail the Perpetual Youth OLG model featured in Section 5.2.

Individuals. Home households solve

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta\phi_i)^t\left\{u(c_{jit})-v(\ell_{jit})\right\}\right] \tag{121}$$

subject to

$$c_{jit}+a_{jit}=(1-\tau_\ell)w_t\ell_{jit}+\tau_{jit}+z_{jit}+\frac{1+i_{t-1}}{(1+\pi_t)\cdot\phi_i}a_{jit-1}\,. \tag{122}$$

Furthermore, hand-to-mouth individuals simply consume their income and do not borrow or save. Symmetrically, Foreign OLG individuals maximize,

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta\phi_i)^t\left\{u(c^*_{jit})-v(\ell^*_{jit})\right\}\right] \tag{123}$$

subject to

$$c^*_{jit}+a^*_{jit}=(1-\tau^*_\ell)w^*_t\ell^*_{jit}+\tau^*_{jit}+z^*_{jit}+\frac{1+i^*_{t-1}}{(1+\pi^*_t)\cdot\phi_i}a^*_{jit-1}\,. \tag{124}$$

Families. The linearized consumption function of the family can be expressed into an Euler Equation representation of aggregate demand. For the Home member, it is

$$\begin{aligned}&[1-\phi_i(1-\beta\phi_i)]\widehat{c}_{it}-\beta\phi_i\widehat{c}_{it+1}-(1-\beta\phi_i)(1-\phi_i)\left(\beta^{-1}\widehat{a}_{it-1}+\frac{\overline{QY}^*}{\overline{C}_i}\widehat{\omega}^*_{it}\right)\\&=(1-\beta\phi_i)(1-\phi_i)\left\{\widehat{e}_{it}+\frac{\overline{A}_i}{\overline{C}_i}\widehat{r}_{t-1}+\frac{\overline{Q\omega}^*_i}{\overline{C}_i}\widehat{q}_t\right\}-\left(\varphi_i+(1-\beta\phi_i)\beta\phi_i\frac{\overline{A}_i}{\overline{C}_i}\right)\widehat{r}_t\,,\end{aligned} \tag{125}$$

and for the Foreign member, it is

$$\begin{aligned}&[1-\phi_i(1-\beta\phi_i)]\widehat{c}^*_{it}-\beta\phi_i\widehat{c}^*_{it+1}-(1-\beta\phi_i)(1-\phi_i)\left(\beta^{-1}\widehat{a}^*_{it-1}-(1+\tau^*_\omega)\frac{\overline{Y}^*}{\overline{C}^*_i}\widehat{\omega}^*_{it}\right)\\&=(1-\beta\phi_i)(1-\phi_i)\left\{\widehat{e}^*_{it}+\frac{\overline{A}^*_i}{\overline{C}^*_i}\widehat{r}^*_{t-1}\right\}-\left(\varphi^*_i+(1-\beta\phi_i)\beta\phi_i\frac{\overline{A}^*_i}{\overline{C}^*_i}\right)\widehat{r}^*_t\,.\end{aligned} \tag{126}$$

We aggregate budget constraints within types which results in the following (linearized) budget constraints,

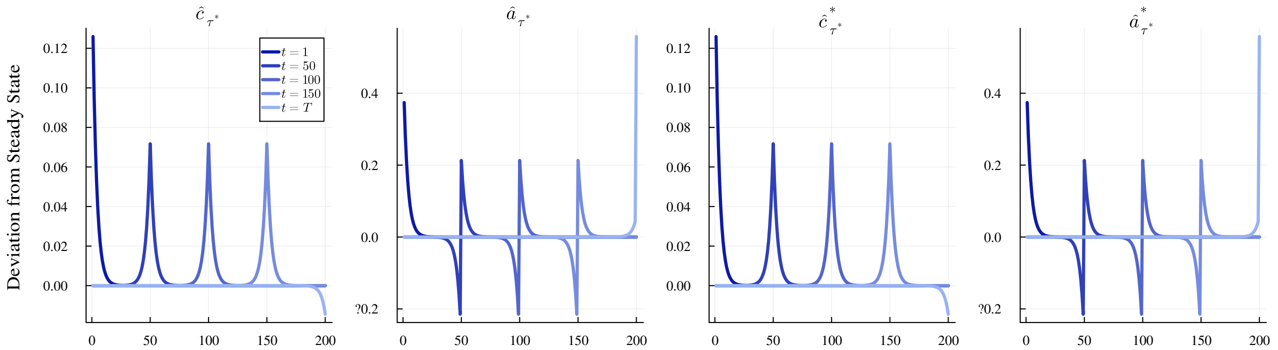
$$\widehat{c}_{it}+\widehat{a}_{it}-\beta^{-1}\widehat{a}_{it-1}-\frac{\overline{QY}^*}{\overline{C}_i}\widehat{\omega}^*_{it}=\widehat{e}_{it}+\frac{\overline{A}_i}{\overline{C}_i}\widehat{r}_{t-1}+\frac{\overline{Q\omega}^*_i}{\overline{C}_i}\widehat{q}_t\,, \tag{127}$$

$$\widehat{c}^*_{it}+\widehat{a}^*_{it}-\beta^{-1}\widehat{a}^*_{it-1}+(1+\tau^*_\omega)\widehat{\omega}^*_{it}=\widehat{e}^*_{it}+\frac{\overline{A}^*_i}{\overline{C}^*_i}\widehat{r}^*_{t-1}\,. \tag{128}$$

Finally, the linearized within-household risk-sharing condition is

$$\widehat{c}_{it}-\widehat{c}^*_{it}=\frac{\widehat{q}_t}{\sigma}\,. \tag{129}$$

Figure C.1: Sequence Space Jacobians — Family



Notes: This figure shows select columns of the sequence space Jacobian of the family's problem due to an increase in Foreign transfers.

Since the remittance tax is assumed to be constant, it does not enter in (129). Equations (125),(126),(127), (128), and (129) characterizes the family block (of a given type).

Aggregation. The Home and Foreign aggregate budget constraints are

$$C_t + A_t = (1 - \tau_\ell) w_t \ell_t + d_t + \tau_t + \frac{1 + i_{t-1}}{1 + \pi_t} A_{t-1} + Q_t \omega_t^* \tag{130}$$

$$C_t^* + A_t^* + (1 + \tau_\omega^*)\omega_t^* = (1 - \tau_\ell^*) w_t^* \ell_t^* + d_t^* + \tau_t^* + \frac{1 + i_{t-1}^*}{1 + \pi_t^*} A_{t-1}^* \tag{131}$$

Figure C.1 shows the remainder of the sequence-space Jacobians of consumption and assets with respect to Foreign transfers.

C.4 Linearized Model: Other Blocks

We restate the linearized version of the equations for completeness.

Unions. Following Auclert, Rognlie, and Straub (2024), labor supply is intermediated by unions. There is no sorting between cohorts and unions and unions impose the same hours on each worker. Real wages are $w_t \equiv \frac{W_t}{P_t}$, $w_t^* \equiv \frac{W_t^*}{P_t^*}$, and nominal wage inflation is $1 + \pi_t^w = \frac{w_t}{w_{t-1}}(1 + \pi_t)$, $1 + \pi_t^{w*} = \frac{w_t^*}{w_{t-1}^*}(1 + \pi_t^*)$. Changing nominal wages incurs an adjustment cost as in Rotemberg (1982). This results in a wage-Phillips Curve,

$$\widehat{\pi}_t^w = \kappa_w \left\{ \frac{1}{\varphi} \widehat{\ell}_t + \frac{1}{\sigma} \widehat{c}_t - \left(\widehat{y}_t - \widehat{\ell}_t \right) \right\} + \beta \widehat{\pi}_{t+1}^w \,, \tag{132}$$

where $\kappa_w \equiv \frac{\varepsilon}{\psi} v'(\overline{\ell})\overline{\ell}$ is the slope of the wage-Phillips curve. A symmetric expression holds for the Foreign economy,

$$\widehat{\pi}_t^{w*} = \kappa_{w*} \left\{ \frac{1}{\varphi} \widehat{\ell}_t^* + \frac{1}{\sigma} \widehat{c}_t^* - \left(\widehat{y}_t^* - \widehat{\ell}_t^* \right) \right\} + \beta \widehat{\pi}_{t+1}^{w*} \,. \tag{133}$$

Production and Pricing. There is a linear production technology in each country,

$$\widehat{y}_t = \widehat{\theta}_t + \widehat{\ell}_t \tag{134}$$

and

$$\widehat{y}_t^* = \widehat{\theta}_t^* + \widehat{\ell}_t^* \tag{135}$$

Prices are flexible and there is no pricing to market. The linearized optimal price is

$$(\widehat{p}_{Ht} - \widehat{p}_t) = \widehat{w}_t - \widehat{\theta}_t \ , \tag{136}$$

and under PCP, $\widehat{p}^*_{Ht} = \widehat{p}_{Ht} - \widehat{e}_t$. Home inflation is defined as $\widehat{\pi}_{Ht} \equiv \widehat{p}_{Ht} - \widehat{p}_{Ht-1}$ and under PCP, $\widehat{\pi}^*_{Ht} = \widehat{\pi}_{Ht} - \Delta\widehat{e}_t$. Aggregate inflation is a weighted sum of Home and Foreign inflation, $\widehat{\pi}_t = (1-\gamma)\widehat{\pi}_{Ht} + \gamma\widehat{\pi}_{Ft}$.

$$(\widehat{p}^*_{Ft} - \widehat{p}^*_t) = \widehat{w}^*_t - \widehat{\theta}^*_t \ . \tag{137}$$

Symmetrically, $\widehat{\pi}^*_{Ft} \equiv \widehat{p}^*_{Ft} - \widehat{p}^*_{Ft-1}, \widehat{\pi}_{Ft} = \widehat{\pi}^*_{Ft} + \Delta\widehat{e}_t$, $\widehat{\pi}^*_t = (1-\gamma)\left(\frac{\overline{P}^*_F}{\overline{P}^*}\right)^{-\theta}\widehat{\pi}^*_{Ft} + \gamma\left(\frac{\overline{P}^*_H}{\overline{P}^*}\right)^{-\theta}\widehat{\pi}^*_{Ht}$, and $d^*_t = \int_0^1 d^*_t(k)dk$.

Financial sector. Households in each country save using a pass-through intermediary. The Home intermediary's balance sheet,

$$\widehat{a}_t = \widehat{b}_t + \widehat{\text{nfa}}_t \tag{138}$$

while in Foreign, the balance sheet is,

$$\widehat{a}^*_t = \widehat{b}^*_t + \widehat{\text{nfa}}^*_t \ . \tag{139}$$

No arbitrage implies that the real UIP condition must hold

$$\widehat{r}_t - \widehat{r}^*_t = \Delta\widehat{q}_{t+1} \ . \tag{140}$$

Policy. Monetary policy sets the Home nominal interest rate i_t according to the the following interest rate rules,

$$\widehat{i}_t = \rho_i\widehat{i}_{t-1} + (1-\rho_i)\psi_\pi\widehat{\pi}_t \quad \text{and} \quad \widehat{i}^*_t = \rho_i\widehat{i}^*_{t-1} + (1-\rho_i)\psi^*_\pi\widehat{\pi}^*_t \ . \tag{141}$$

We consider a rule where the government debt returns to the steady state gradually over time. In particular, we specify the following fiscal rule

$$\widehat{b}_t = \rho_b\widehat{b}_{t-1} + \frac{\overline{B}}{\overline{Y}}\widehat{i}_{t-1} - \frac{\overline{B}}{\overline{Y}}(1+\overline{r})\widehat{\pi}_t - \frac{\tau_\ell\overline{w}\overline{\ell}}{\overline{Y}}\left(\widehat{w}_t + \widehat{\ell}_t\right) + \frac{\overline{P}_H}{\overline{P}}\widehat{\tau}^x_t. \tag{142}$$

where $\rho_b \in (0,1)$ captures the speed at which government debt returns to steady state. The Home government budget constraint is,

$$\begin{aligned}\widehat{b}_t + \frac{\tau_\ell\overline{w}\overline{\ell}}{\overline{Y}}\left(\widehat{w}_t + \widehat{\ell}_t\right) &= \frac{\overline{P}_H}{\overline{P}}\left(\widehat{\tau} + \frac{\overline{\tau}}{\overline{Y}}(\widehat{p}_{Ht} - \widehat{p}_t)\right) + \frac{\overline{P}_H}{\overline{P}}\left(\widehat{g}_t + \frac{\overline{G}}{\overline{Y}}(\widehat{p}_{Ht} - \widehat{p}_t)\right) \\ &\quad + (1+\overline{r})\widehat{b}_{t-1} + \frac{\overline{B}}{\overline{Y}}(\widehat{i}_{t-1} - (1+\overline{r})\widehat{\pi}_t) \ .\end{aligned} \tag{143}$$

Note that combining the fiscal rule and the government budget constraint implies that transfers are equal to

$$\widehat{\tau}_t = -\frac{\overline{P}}{\overline{P}_H}(1+\overline{r}-\rho_b)\widehat{b}_{t-1} + \widehat{\tau}^x_t - \widehat{g}_t - \left(\frac{\overline{\tau}+\overline{G}}{\overline{Y}}\right)(\widehat{p}_{Ht} - \widehat{p}_t) \, .$$

We impose a similar fiscal rule in Foreign,

$$\widehat{b}_t^* = \rho_b^* \widehat{b}_{t-1}^* + \frac{\overline{B}^*}{\overline{Y}^*}\widehat{i}_{t-1}^* - \frac{\overline{B}^*}{\overline{Y}^*}(1+\overline{r}^*)\widehat{\pi}_t^* - \frac{\tau_\ell^* \overline{w}^* \overline{\ell}^*}{\overline{Y}^*}\left(\widehat{w}_t^* + \widehat{\ell}_t^*\right) - \tau_\omega^* \widehat{\omega}_t^* + \frac{\overline{P}_F^*}{\overline{P}^*}\widehat{\tau}_t^{x*} , \tag{144}$$

with the Foreign government budget constraint

$$\begin{aligned} \widehat{b}_t^* + \frac{\tau_\ell^* \overline{w}^* \overline{\ell}^*}{\overline{Y}^*}\left(\widehat{w}_t^* + \widehat{\ell}_t^*\right) &= \frac{\overline{P}_F^*}{\overline{P}^*}\left(\widehat{\tau}^* + \frac{\overline{\tau}^*}{\overline{Y}^*}(\widehat{p}_{Ft}^* - \widehat{p}_t^*)\right) + \frac{\overline{P}_F^*}{\overline{P}^*}\left(\widehat{g}_t^* + \frac{\overline{G}^*}{\overline{Y}^*}(\widehat{p}_{Ft}^* - \widehat{p}_t^*)\right) \\ &+ (1+\overline{r}^*)\widehat{b}_{t-1}^* + \frac{\overline{B}^*}{\overline{Y}^*}(\widehat{i}_{t-1}^* - (1+\overline{r}^*)\widehat{\pi}_t^*) . \end{aligned} \tag{145}$$

and

$$\widehat{\tau}_t^* = -\frac{\overline{P}^*}{\overline{P}_F^*}(1+\overline{r}^* - \rho_b^*)\widehat{b}_{t-1}^* + \widehat{\tau}_t^{x*} - \widehat{g}_t^* - \left(\frac{\overline{\tau}^* + \overline{G}^*}{\overline{Y}^*}\right)(\widehat{p}_{Ft}^* - \widehat{p}_t^*) .$$

Market Clearing and Equilibrium. There are five market clearing conditions. One each for Home and Foreign goods, and similarly for the labor market in each country.[24] Finally, there is a world asset market clearing condition. We work with the symmetric steady state where $\overline{Q} = 1$, $\overline{\omega}^* = 0$, $\overline{Y} = \overline{Y}^*$, and initial net foreign assets are zero. That is, $\overline{A} = \overline{B}$, $\overline{A}^* = \overline{B}^*$. The goods market clearing conditions are

$$\widehat{y}_t = \frac{\overline{C}_H}{\overline{Y}}\widehat{c}_{Ht} + \frac{\overline{C}_H^*}{\overline{Y}}\widehat{c}_{Ht}^* + \frac{\overline{G}}{\overline{Y}}\widehat{g}_t , \tag{146}$$

and

$$\widehat{y}_t^* = \frac{\overline{C}_F^*}{\overline{Y}^*}\widehat{c}_{Ft}^* + \frac{\overline{C}_F}{\overline{Y}^*}\widehat{c}_{Ft} + \frac{\overline{G}^*}{\overline{Y}^*}\widehat{g}_t^* . \tag{147}$$

It will also be useful to track net exports. The linearized expression for net exports are

$$\widehat{nx}_t = \frac{\gamma(2\theta(1-\gamma)-1)}{1-2\gamma}\widehat{q}_t + \gamma(\widehat{c}_t^* - \widehat{c}_t), \quad \text{and} \quad \widehat{nx}_t^* = \frac{\gamma(1-2\theta(1-\gamma))}{1-2\gamma}\widehat{q}_t + \gamma(\widehat{c}_t - \widehat{c}_t^*) , \tag{148}$$

where $\widehat{nx}_t \equiv \frac{NX_t - \overline{NX}}{\overline{Y}}$ and $\widehat{nx}_t^* \equiv \frac{NX_t^* - \overline{NX}^*}{\overline{Y}^*}$.[25] Note that $\widehat{nx}_t + \widehat{nx}_t^* = 0$. Moreover, linearizing the current account equations, we have

$$\widehat{ca}_t = \overline{r}\widehat{\mathrm{nfa}}_{t-1} + \widehat{nx}_t + \widehat{\omega}_t^* \tag{149}$$

$$\widehat{ca}_t^* = \overline{r}^*\widehat{\mathrm{nfa}}_{t-1}^* + \widehat{nx}_t^* - \widehat{\omega}_t^* \tag{150}$$

[24]Formally, the labor market does not clear due to the union setup. However, the implied labor supply from the union block must be consistent with the labor demand from the firm block.

[25]The general expressions for net exports are

$$\widehat{nx}_t = \frac{\overline{\mathcal{E}}\overline{P}_H^*\overline{C}_H^*}{\overline{P}\overline{Y}}(\widehat{e}_t + \widehat{p}_{Ht}^* - \widehat{p}_t + \widehat{c}_{Ht}^*) - \frac{\overline{P}_F\overline{C}_F}{\overline{P}\overline{Y}}(\widehat{p}_{Ft} - \widehat{p}_t + \widehat{c}_{Ft})$$

$$\widehat{nx}_t^* = \frac{\overline{\mathcal{E}}^{-1}\overline{P}_F\overline{C}_F}{\overline{P}^*\overline{Y}^*}(\widehat{p}_{Ft} - \widehat{e}_t - \widehat{p}_t^* + \widehat{c}_{Ft}) - \frac{\overline{P}_H^*\overline{C}_H^*}{\overline{P}^*\overline{Y}^*}(\widehat{p}_{Ht}^* - \widehat{p}_t^* + \widehat{c}_{Ht}^*)$$

for which we substitute in the openness share, $\gamma = \gamma^*$, along with the real exchange rate, $\widehat{q}_t = \widehat{e}_t + \widehat{p}_t^* - \widehat{p}_t$ and terms-of-trade, $\widehat{s}_t = \widehat{p}_{Ft} - \widehat{p}_{Ht}^* - \widehat{e}_t = \widehat{p}_{Ft}^* + \widehat{e}_t - \widehat{p}_{Ht}$. Under this case, $\widehat{s}_t = \frac{1}{1-2\gamma}\widehat{q}_t$. Combining with the linearized demand equations, we get the expression in the text.

where $\widehat{b}_t \equiv \frac{B_t - \overline{B}}{\overline{Y}}$, $\widehat{b}^*_t \equiv \frac{B^*_t - \overline{B}^*}{\overline{Y}^*}$.[26] The linearized world asset market clearing condition is given by

$$\widehat{\text{nfa}}_t + \widehat{\text{nfa}}^*_t = 0 \,. \tag{151}$$

Goods and asset market clearing conditions imply $\widehat{ca}_t + \widehat{ca}^*_t = 0$. That is, goods, assets and remittances must net out to zero at the world level.

Definition (Competitive Equilibrium). *Let $\widehat{\boldsymbol{u}} \equiv \{\widehat{u}_t\}_{t=0}^{\infty}$. A (sequential) competitive equilibrium is defined as a set of policies $\{\widehat{\boldsymbol{\tau}}, \widehat{\boldsymbol{\tau}}^*, \widehat{\boldsymbol{i}}, \widehat{\boldsymbol{i}}^*, \widehat{\boldsymbol{b}}, \widehat{\boldsymbol{b}}^*\}$, prices $\{\widehat{\boldsymbol{\pi}}, \widehat{\boldsymbol{\pi}}^*, \widehat{\boldsymbol{\pi}}^W, \widehat{\boldsymbol{\pi}}^{W*}, \widehat{\boldsymbol{p}}_H, \widehat{\boldsymbol{p}}^*_H, \widehat{\boldsymbol{p}}_F, \widehat{\boldsymbol{p}}^*_F, \widehat{\boldsymbol{e}}, \widehat{\boldsymbol{q}}, \widehat{\boldsymbol{r}}, \widehat{\boldsymbol{r}}^*\}$ and quantities $\{\widehat{\boldsymbol{\omega}}^*, \widehat{\boldsymbol{y}}, \widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{c}}, \widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{c}}_H, \widehat{\boldsymbol{c}}^*_H, \widehat{\boldsymbol{c}}_F, \widehat{\boldsymbol{c}}^*_F, \widehat{\boldsymbol{\ell}}, \widehat{\boldsymbol{\ell}}^*, \widehat{\boldsymbol{d}}, \widehat{\boldsymbol{d}}^*\}$ such that:*

1. Households: Given prices and incomes, $\{\widehat{\boldsymbol{c}}, \widehat{\boldsymbol{c}}^*, \widehat{\boldsymbol{b}}, \widehat{\boldsymbol{b}}^*, \widehat{\boldsymbol{\omega}}^*\}$ solves the household's problems.
2. Unions: $\{\widehat{\boldsymbol{\pi}}^W, \widehat{\boldsymbol{\pi}}^{W*}\}$ are consistent with the wage-Phillips curves (132) and (133)
3. Firms: $\{\widehat{\boldsymbol{p}}_H, \widehat{\boldsymbol{p}}^*_F, \widehat{\boldsymbol{y}}, \widehat{\boldsymbol{y}}^*, \widehat{\boldsymbol{d}}, \widehat{\boldsymbol{d}}^*\}$ solve the firm's problem, and $\{\widehat{\boldsymbol{p}}^*_H, \widehat{\boldsymbol{p}}_F\}$ are consistent with PCP.
4. Financial intermediaries: The real UIP condition (140) holds.
5. Policies: Given inflation, nominal interest rates $\{\widehat{\boldsymbol{i}}, \widehat{\boldsymbol{i}}^*\}$ are consistent with monetary policy rules (141). $\{\widehat{\boldsymbol{\tau}}, \widehat{\boldsymbol{b}}\}$ and $\{\widehat{\boldsymbol{\tau}}^*, \widehat{\boldsymbol{b}}^*\}$ are consistent with the fiscal rules and government budget constraints in Home and Foreign respectively.
6. Market clearing: The goods market clearing (146) and (147) holds, the labor market clears such that $\{\widehat{\boldsymbol{\ell}}, \widehat{\boldsymbol{\ell}}^*\}$ are consistent with the production technologies (134) and (135) and the wage NKPCs (132), (133). The world asset market (151) clears.

C.5 Derivations - Household Block

C.5.1 Linearized Consumption Function

Let $\widehat{p}_{Ht} \equiv d \log P_{Ht}$, $\widehat{p}^*_{Ht} \equiv d \log P^*_{Ht}$, and $\widehat{p}_t \equiv d \log P_t$. Linearizing, we have

$$\widehat{c}_{Ht} = -\theta(\widehat{p}_{Ht} - \widehat{p}_t) + \widehat{c}_t \,, \tag{152}$$

$$\widehat{c}_{Ft} = -\theta(\widehat{p}_{Ft} - \widehat{p}_t) + \widehat{c}_t \,, \tag{153}$$

$$\widehat{p}_t = (1-\gamma)\left(\frac{\overline{P}_H}{\overline{P}}\right)^{1-\theta} \widehat{p}_{Ht} + \gamma \left(\frac{\overline{P}_F}{\overline{P}}\right)^{1-\theta} \widehat{p}_{Ft} \,, \tag{154}$$

$$\widehat{p}^*_t = (1-\gamma^*)\left(\frac{\overline{P}^*_F}{\overline{P}^*}\right)^{1-\theta} \widehat{p}^*_{Ft} + \gamma^* \left(\frac{\overline{P}^*_H}{\overline{P}^*}\right)^{1-\theta} \widehat{p}^*_{Ht} \,. \tag{155}$$

[26] The full expression for the current accounts are

$$\widehat{ca}_t \equiv \widehat{\text{nfa}}_t - \widehat{\text{nfa}}_{t-1} = \overline{r}\widehat{\text{nfa}}_{t-1} + \frac{\overline{\text{nfa}}}{\overline{Y}}\widehat{r}_{t-1} + \widehat{nx}_t + \frac{\overline{QY}^*}{\overline{Y}}\widehat{\omega}^*_t + \frac{\overline{Q\omega}^*}{\overline{Y}}\widehat{q}_t \,,$$

$$\widehat{ca}^*_t \equiv \widehat{\text{nfa}}^*_t - \widehat{\text{nfa}}^*_{t-1} = \overline{r}\widehat{\text{nfa}}^*_{t-1} + \frac{\overline{\text{nfa}}^*}{\overline{Y}^*}\widehat{r}^*_{t-1} + \widehat{nx}^*_t - \widehat{\omega}^*_t \,.$$

Since net foreign assets and remittances are zero in steady state, there are no valuation effects resulting from real interest rate and exchange rate fluctuations. This simplifies the current account equations to the expression in the main text.