Global Value Chains and Inflation Dynamics: Does the Source of Inputs Matter?*

Tommaso Aquilante †   Aydan Dogan ‡   Melih Firat §   Aditya Soenarjo ¶

January 2026

Abstract

This paper studies how Global Value Chains (GVCs) shape inflation dynamics in open economies. Using industry-level data for the UK, we find that the source-country composition of imported intermediates is critical: greater reliance on inputs sourced from emerging market economies significantly weakens the inflation-output link where the integration has risen most sharply in highly tradable capital-goods industries. By contrast, GVCs integration with advanced economies does not significantly affect the inflation-output relationship. To interpret these findings, we develop a two-country, multi-sector New Keynesian model with trade in intermediate and final goods. The model shows that sectoral openness and substitution between domestic and foreign intermediates govern the transmission of shocks to imported input prices. In highly open sectors, higher foreign input costs induce substitution toward domestic inputs, generating demand-like responses despite rising marginal costs, whereas less open sectors contract through income effects. These mechanisms explain how the sector-source dimension of GVCs integration reshapes the inflation-output relationship in advanced open economies.

Keywords: Global value chains, inflation dynamics, Phillips curve.
JEL codes: E30, E31, E32, F41, F44.

*The views expressed in this paper are solely those of the authors and so cannot be taken to represent those of the Bank of England and the IMF, its Executive Board, or IMF management. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Boston or the Federal Reserve System. For helpful comments and suggestions, we thank Laura Alfaro, Ravi Balakrishnan, Laurence Ball, Ambrogio Cesa-Bianchi, Diego Comin, Giancarlo Corsetti, Federico Di Pace, Sophie Guilloux-Nefussi, Simon Gilchrist, Alex Haberis, Ida Hjortsoe, Simon Lloyd, Riccardo Masolo, Ricardo Reis, Rana Sajedi, Ayşegül Şahin, Boromeus Wanengkirtyo, Jonathan Wright and Robert Zymek. We are also grateful to participants at numerous seminars and conferences including at the Bank of England, Johns Hopkins University, LSE and the NY FED. We thank Veronica Veggente and Jack Terry for excellent research assistance.

†Tommaso Aquilante was previously employed by the Bank of England while working on this paper. E-mail: aquilantetom@gmail.com

‡Bank of England. E-mail: aydan.dogan@bankofengland.co.uk

§IMF. E-mail: mfirat@imf.org

¶Federal Reserve Bank of Boston. E-mail: adityasoenarjo@gmail.com

1 Introduction

In recent years, there has been renewed interest in how the globalization of production shapes inflation dynamics. Rising trade fragmentation and the resurgence of protectionist policies have brought Global Value Chains (GVCs) to the forefront of macroeconomic discussions, raising a central question: how do changes in GVC participation affect the relationship between inflation and output? More specifically, does reliance on imported intermediate inputs—especially from emerging market economies rather than advanced economies—affect the Phillips curve dynamics? This paper examines the role of imported inputs in shaping inflation dynamics and assesses whether the source-country of those inputs matters for the strength of the link between inflation and domestic economic activity.

A key motivation for our analysis is that GVC participation has not only increased but has evolved unevenly across both sectoral and source-country dimensions. Figure 1 documents this pattern for the UK, an open advanced economy: capital-goods sectors (e.g., computer electronics) experienced sizable increases in intermediates sourced from emerging market economies (EMEs), while their reliance on inputs from advanced economies (AEs) remained broadly stable. In contrast, service sectors (e.g., hospitality) display little change in imported-input shares from either source. This sector-source asymmetry indicates that changes in GVC integration are far from uniform, motivating our analysis of how these changes shape inflation dynamics and the inflation-output gap relationship.

We begin by developing a static, two-country model with input-output linkages in a roundabout production structure. We show that the share of imported intermediate inputs affects the Phillips curve relationship through two distinct channels: (i) a direct cost channel, whereby a higher foreign-input share dampens the sensitivity of marginal costs to domestic activity shocks, and therefore flattens the Phillips curve; and (ii) a substitution channel, whereby the elasticity of substitution between domestic and foreign intermediates governs the pass-through of international relative-price movements into production costs. We then extend the model to a multi-sector setting and derive a sectoral Phillips curve relationship, which we subsequently take to the data.

Guided by the model's prediction that imported input dependence affects the inflation-output relationship, we first examine whether sectoral variation in imported intermediate goods shares helps explain cross-sector differences in the Phillips curve in the UK. Using industry-level data from the World Input-Output Database (WIOD) over 2000-2014, we find that changes in the overall imported input share at the sector level do not significantly affect the inflation–output gap relationship. However, once we exploit variation along the source dimension, distinguishing intermediates sourced from AEs and EMEs, a clear pattern emerges: increases in intermediates sourced from EMEs significantly weaken the inflation-output link, whereas changes in AE integration do not.

Together, these results point to a previously underexplored dimension of GVCs, variation across both sectors and source countries, that is central to understanding inflation dynamics in open economies. Reliance on EME-sourced intermediates is concentrated in capital-goods industries, suggesting that changes in imported input dependence may transmit differently across sectors. To interpret these patterns, we develop a dynamic, multi-sector, two-country New Keynesian model with trade in intermediate and final goods. Unlike the static framework, the quantitative model captures propagation through production networks as well as interactions with nominal rigidities and monetary policy. Calibrated to AE and EME data for services, manufacturing, and capital goods, the model allows us to shed light on the source-specific mechanisms behind our empirical findings.

We use the model to study sector-specific tariff shocks that raise the relative price of foreign intermediate inputs and thereby reduce imported input shares. While these shocks generate similar aggregate responses—higher inflation and lower output and real wages—their sectoral effects differ significantly. In particular, shocks to the capital-goods sector generate demand-like dynamics, with output expanding despite higher costs. Shocks to manufacturing instead generate a more standard supply-driven response, whereas shocks to services lead to contractions in both output and inflation, reflecting strong income effects in a largely non-traded sector. These heterogeneous responses reflect differences in sectoral openness and positions within the production network.

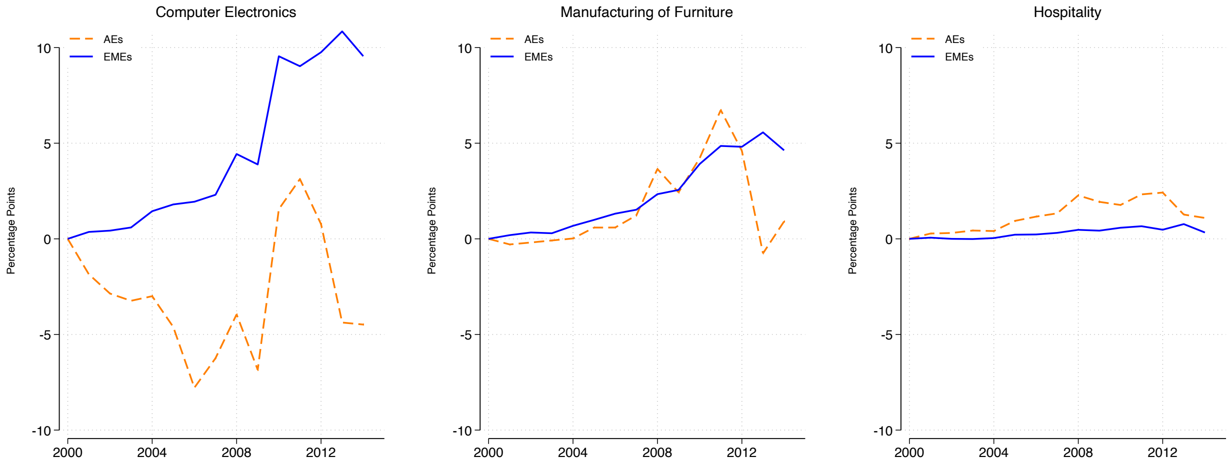
Counterfactual experiments clarify the underlying mechanisms. When cross-sector input-output linkages are suppressed, sector-specific shocks behave as standard cost-push disturbances as highlighted in Werning et al. (2025), raising inflation and lowering output across all sectors. Allowing for substitution between domestic and foreign intermediates reintroduces heterogeneity: highly open sectors can substitute toward domestic inputs when imported inputs become more expensive, generating expansionary responses despite higher marginal costs, while less-open sectors contract as a result of fall in domestic demand. Thus, the interaction between sectoral openness, substitution elasticities, and production networks is central to understanding the transmission of GVC shocks.

These mechanisms provide a unified interpretation of our empirical findings. GVCs integration with EMEs has increased most sharply in highly tradable capital goods sectors, where changes in the price or availability of foreign intermediate inputs trigger large reallocations of input demand. In both the data and the model, the share of imported intermediate inputs responds endogenously to international relative prices. When reliance on EME sourced intermediates rises in these sectors, shocks to foreign input prices and availability have stronger effects on domestic marginal costs and demand. As a result, the inflation-output relationship in advanced open economies is affected by the sector and source composition of GVCs integration.

Related Literature. Our paper builds on the large literature examining how globalization influences inflation dynamics. Existing work primarily focuses on the depth of trade integration—overall openness, import penetration, or participation in global value chains—abstracting from heterogeneity in the composition of trade relationships. In contrast, we show that the sector-source composition of imported intermediate inputs, particularly integration with emerging versus advanced economies, is central to understanding how GVCs affect the inflation-output relationship across sectors.

Previous work has highlighted the role of global factors in shaping the sensitivity of inflation to domestic slack (e.g. Auer and Fischer, 2010; Bianchi and Civelli, 2015; Forbes, 2019; Guerrieri et al., 2010; Guilloux-Nefussi, 2020; Heise et al., 2022; Obstfeld, 2020). Our empirical strategy is related to Gilchrist and Zakrajsek (2019), who use industry-level data to show that greater trade integration contributed to the decline in the sensitivity of U.S. inflation to the domestic output gap. We also rely on industry-level data but depart from their approach in two key respects. First, rather than studying overall trade integration, which combines final and intermediate goods, we focus specifically on imported intermediates, isolating the GVC dimension of openness. Second, while most empirical work is U.S.-focused and abstracts from the rise of EMEs in global production, we document the

Figure 1: Change in Imported Intermediate Goods Share



Note: Source - WIOD. Figure plots the change in the shares of imported intermediate goods for selected sectors from advanced economies (AEs) and emerging market economies (EMEs). Country classifications follow IMF and details are provided in Appendix A.

importance of the source-sector dimension for understanding inflation dynamics in an open economy like the UK.

Our modeling strategy is closest to Comin and Johnson (2020), who develop a small-open-economy New Keynesian model with trade in intermediate and final goods to study the effects of a permanent increase in trade openness on U.S. inflation. In contrast, we study sector-specific tariff shocks that reduce imported input shares in a two-country framework. Instead of focusing on overall inflation, we examine how these shocks shape inflation and output dynamics, emphasizing how sectoral production-network positions influence transmission. Our work is also related to Rubbo (2023), who show in a closed economy that intermediate input use flattens the Phillips curve. Using a related multi-sector structure in an open economy, we show how trade in intermediates can generate similar effects.

A growing literature studies shocks to GVCs, particularly in the context of post-pandemic inflation (e.g. Amiti et al., 2023; Di Giovanni et al., 2023). Rather than explaining a specific inflation episode, we focus on the broader question of how shocks to GVCs integration shape inflation dynamics in open economies. In this respect, our paper relates to recent work on trade fragmentation. Using a two-sector small open economy New Keynesian model, Ambrosino et al. (2024) examine various fragmentation scenarios and show that fragmentation may not be inflationary if households anticipate it and adjust demand accordingly.[1] We differ in focus and mechanism: rather than studying anticipatory behavior or terms-of-trade shocks, we analyze how sector-specific reductions in imported-input shares propagate through production networks and affect the inflation and output dynamics, emphasizing the role of sectoral openness and substitutability.

Closely related, Kalemli-Özcan et al. (2025) develop a New Keynesian open-economy model with input-output linkages to examine how tariff-induced distortions propagate through global produc-

[1] There is also a large and expanding literature on optimal policy in the presence of tariffs, including Auray et al. (2025); Bergin and Corsetti (2025, 2023); Bianchi and Coulibaly (2025); Werning et al. (2025), among others. Although we also study the transmission of GVC de-integration through tariff shocks, our objective is not to derive optimal policy. Instead, we highlight how sectoral heterogeneity shapes the propagation of sector-specific tariff shocks.

tion chains and shape the behavior of inflation. Like us, they emphasize the importance of sectoral production linkages and shocks to imported intermediates. Our contribution is complementary: we provide new empirical evidence on sector-source heterogeneity in the Phillips curve, and we develop a two-country, multi-sector quantitative model designed to uncover the mechanism rooted in the prominence of EME-sourced intermediates in capital-goods sector that drives our empirical findings.

Roadmap. Section 2 introduces the static model of global value chains, which lays out how trade in intermediate inputs affect the Phillips curve relationship. Section 3 presents the empirical analysis and a set of robustness checks. Section 4 presents the multi-sector, two-country DSGE model, its calibration, and the counterfactual exercises that uncover the mechanisms behind the empirical results. Section 5 concludes.

2 A Model of Global Value Chains

This section introduces a static model with GVCs. We begin with a two-country economy with roundabout production in which Home (H) and Foreign (F) differ in size, with population shares n and $1-n$, respectively. The model predicts how a country's integration to the GVCs affects the Phillips curve relationship. Next, we extend the model to include multiple sectors. The introduction of multiple sectors allow us to formulate a sectoral producer price index (PPI) Phillips curve, which we use to motivate our empirical specification.

2.1 Roundabout Production with Imported Intermediate Inputs

In this section, we analyze how GVC integration affects the slope of the Philips Curve. We build on the work of Rubbo (2023) to derive a theoretical relationship between GVC integration and the Phillips curve. Here we simplify our model by abstracting from multiple sectors and focusing on one period only as we are interested in how the use of imported intermediate inputs affects the slope of the Phillips curve. Throughout the paper, we use the notation "$*$" to capture variables in the foreign economy.

Households. Households in each country consume Home and Foreign goods according to a general consumption aggregator,

$$C = \mathcal{C}\left(C_H, C_F\right) \ , \tag{1}$$

and supply labor to firms under standard iso-elastic preferences,

$$u(C,L) = \frac{C^{1-\sigma}}{1-\sigma} - \Xi \frac{L^{1+\varphi}}{1+\varphi} \ , \tag{2}$$

where σ and φ denote the inverse of inter-temporal elasticity of substitution and Frisch elasticity of labor supply, respectively, and Ξ scales the disutility of labor.

Production. Firms in each economy are identical and use labor (L) and intermediate inputs (M) to produce a unit of output. Production has a round-about structure. The production function has the following constant-returns-to-scale functional form

$$Y_H(i) = AL(i)^{\delta} M(i)^{1-\delta}\,, \tag{3}$$

where Y_H denotes firm i's gross-output of home goods, A is the aggregate productivity and δ denotes the share of labor in production. Intermediate goods used by the firms are a CES aggregate of home and foreign-produced intermediate inputs

$$M(i) = \left[\mu^{\frac{1}{\phi}} (M_H(i))^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} (M_F(i))^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}\,, \tag{4}$$

where $M_H(i)$ and $M_F(i)$ denote the demand for domestically and foreign-produced intermediate goods, respectively and ϕ denotes the elasticity of substitution between home and foreign-produced intermediate goods. The parameter μ captures home-bias in intermediate inputs. As in De Paoli (2009), we assume that the share of imported is is a function of relative country size, $1-n$, and the degree of openness in production, v_M: $1-\mu = (1-n)\, v_M$.

Pricing. To introduce a Phillips Curve into the model, we allow for nominal rigidities in the form of sticky information as in Mankiw and Reis (2002). The timing within the period is as follows. 1. All firms pre-set their price as a markup over the expected marginal cost. 2. A fraction $1-\theta$ of firms observe the aggregate shocks in the economy and update their price. As such, Home and Foreign firms pre-set their price to a markup over their expected marginal costs,

$$P_H^{\#}(i) = \frac{\epsilon}{\epsilon - 1}\mathbb{E}[MC] \;\text{ and }\; P_F^{*\#}(i) = \frac{\epsilon}{\epsilon-1}\mathbb{E}[MC^*]\,, \tag{5}$$

where the expectation is taken over aggregate states. A fraction $1-\theta$ of Home firms observe the realization of aggregate shocks and hence update their price. These firms change their prices to charge a markup over their actual marginal costs,

$$\widetilde{P}_H(i) = \frac{\epsilon}{\epsilon-1} MC \;\text{ and }\; \widetilde{P}_F^*(i) = \frac{\epsilon}{\epsilon-1} MC^*\,, \tag{6}$$

where $MC = \left(\frac{W}{\delta}\right)^{\delta}\left(\frac{P^M}{1-\delta}\right)^{1-\delta}$. The aggregate price level at the end of the period is given by $P_H^{1-\epsilon} = \theta P_H^{\#1-\epsilon} + (1-\theta)\tilde{P}_H^{1-\epsilon}$ and inflation is defined as the change in prices relative to the pre-set price before any shocks hit the economy. Inflation occurs when the actual marginal cost rises above the expected marginal cost and is approximated to first order by, [2]

$$\pi_H \equiv d\log P_H = (1-\theta) d\log MC\,. \tag{7}$$

A symmetric expression holds for the foreign economy to define π_F^*. We assume producer currency pricing as the baseline.

The Global Phillips Curve. We can relate a firm's pricing decisions to a measure of real activity in each country. Let $\log \mathbf{p} = \begin{pmatrix}\log P_H & \log P_F^*\end{pmatrix}'$ denote a vector of prices of Home and Foreign goods,

[2]Formally, $d\log P_H \equiv \log P_H - \log P_H^{\#}$ and $d\log MC \equiv \log MC - \log \mathbb{E}[MC]$.

$\log \mathbf{A} = \left(\log A \ \ \log A^*\right)'$ denote a vector of TFP levels. Also define three matrices,

$$\Phi = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\alpha^* & \alpha^* \end{pmatrix}, \quad \Omega = \begin{pmatrix} \omega^H & \omega^F \\ \omega^{H*} & \omega^{F*} \end{pmatrix}, \quad \Theta = \text{diag}(1-\theta, 1-\theta^*) \ ,$$

where Φ is the matrix of consumption weights such that the consumer price index is $\log \boldsymbol{P} = \Phi \log \boldsymbol{p}$. Ω represents the global input-output matrix and Θ is the matrix of the frequency of price adjustment. The elements of Ω include the domestic intermediate input share

$$\omega^H = \frac{P^H M^H}{\text{MC} \cdot Y} = \frac{1-\delta}{1 + \left(\frac{1-\mu}{\mu}\right)\left(\bar{P}^{\text{TOT}}\right)^{\phi-1}} \ , \tag{8}$$

and the imported intermediate input share

$$\omega^F = \frac{P^F M^F}{\text{MC} \cdot Y} = \frac{1-\delta}{1 + \left(\frac{\mu}{1-\mu}\right)\left(\bar{P}^{\text{TOT}}\right)^{1-\phi}} \ , \tag{9}$$

where $\bar{P}^{\text{TOT}} = \frac{\bar{P}^H}{\bar{P}^F}$ denotes the steady-state terms-of-trade (expressed in the same currency). ω^{H*} and ω^{F*} are define symmetrically. Note that the intermediate input shares are mechanically related to the home-bias parameters, the labor share and the terms-of-trade. The relationship with the elasticity of substitution is more subtle. If either $\phi = 1$ (Cobb-Douglas) or $\bar{P}^{\text{TOT}} = 1$, then steady-state intermediate input shares do not depend on the elasticity of substitution. In particular, if the two countries are symmetric, this is a sufficient condition for the steady-state terms-of-trade to be equal to 1. The results are summarized in the following lemma.

Lemma 1. *If the elasticity of substitution $\phi \neq 1$ and the steady-state terms-of-trade $\bar{P}^{TOT} \neq 1$, then the domestic and imported intermediate input shares varies with the elasticity of substitution.*

The proof follows from computing the derivatives,

$$\frac{\partial \omega^H}{\partial \phi} = -(1-\delta)\left(\frac{1-\mu}{\mu}\right)\frac{\log(\bar{P}_{\text{TOT}})\bar{P}_{\text{TOT}}^{\phi-1}}{\left[1 + \frac{1-\mu}{\mu}\bar{P}_{\text{TOT}}^{\phi-1}\right]^2}, \quad \frac{\partial \omega^F}{\partial \phi} = (1-\delta)\left(\frac{\mu}{1-\mu}\right)\frac{\log(\bar{P}_{\text{TOT}})\bar{P}_{\text{TOT}}^{1-\phi}}{\left[1 + \frac{\mu}{1-\mu}\bar{P}_{\text{TOT}}^{1-\phi}\right]^2} \ .$$

Intuitively, if $\bar{P}^{TOT} > 1$, it means that Home goods are more expensive than Foreign goods. Therefore, an increase in the elasticity of substitution makes it easier for firms to substitute away from the more expensive good. As a result, the domestic intermediate share decreases and the imported intermediate share increases. If $\bar{P}^{TOT} < 1$, it means the Foreign goods are more expensive than Home goods and we get the opposite result where the domestic intermediate share increases and the foreign intermediate share decreases.

One way to model changes to global value chain integration is by changing taxes on imports. In this case, $\bar{P}^{\text{TOT}} = \frac{\bar{P}^H}{(1+\tau)\bar{P}^F} < 1$. Introducing such a tax makes it more likely that we are in the second case. Now we can proceed with the expression of the Global Phillips Curve.

Proposition 1. *The Global (CPI) Phillips Curve is,*

$$d \log \mathbf{P} = \mathcal{K}\, \tilde{\mathbf{y}} + \mathcal{G}\, d \log \mathbf{A} + \mathcal{H}\, d \log \mathcal{E}, \tag{10}$$

where $\mathcal{K} = \Phi\Theta(\mathbf{I} - \Omega\Theta)^{-1}\delta \left[\mathbf{I} - ((\mathbf{1} + \sigma\Phi - \sigma\mathbf{I})\Theta(\mathbf{I} - \Omega\Theta)^{-1}\delta\right]^{-1} (\sigma + \varphi)$ *is the slope of the Phillips Curve and* $\mathcal{E}$ *is the nominal exchange rate (units of foreign currency in home currency).*

The proof and expressions for the $\mathcal{G}, \mathcal{H}$ matrices are shown in Appendix C.1. This expression shows that inflation dynamics are driven by: (i) domestic and foreign output gap, (ii) cross-country relative productivity, and (iii) exchange rate fluctuations. The main diagonal of $\mathcal{K}$ represents the *slope* of the Phillips Curve – the dependence of CPI inflation on the domestic output gap. The off-diagonal elements of $\mathcal{K}$ capture the dependence of domestic inflation on the foreign output gap.

The matrix $(\mathbf{I} - \Omega\Theta)^{-1}$ captures the adjusted Leontief inverse similar to Rubbo (2023). The key difference is that we emphasize the role of intermediate input shares in decoupling inflation from the (domestic) output gap whereas Rubbo (2023) stresses the role of price-stickiness along the production network. This leads us to the following corollary.

Proposition 2. *The higher the imported intermediate good share, the lower is the slope of the Phillips curve. That is,*

$$\frac{\partial \mathcal{K}_{ii}}{\partial \Omega_{ii}} > 0\,.$$

The proof follows directly from computing the derivative. Intuitively, as firms depend more on intermediate inputs imported from abroad, their marginal costs are less exposed to the domestic output and more exposed to foreign output. As a result, inflation depends less on the domestic output gap and more on the foreign output gap. Given that the share of imported goods (both final demand and intermediate) is proportional to the country size, the relative country size will matter for the slope through α and μ. Smaller countries like the UK are more open, so all else equal should have a flatter Phillips Curve. In addition, unsurprisingly, the Phillips Curve become steeper as labor share increases, consistent with the standard three-equation closed-economy New Keynesian model.

In addition, the price stickiness of domestic and foreign goods, captured by Θ, is also amplified along the production network. The price stickiness of foreign goods implies that the cost of imported intermediate goods, and hence the marginal costs for home firms, do not rise by as much as in the flexible-price case. This then implies that domestic prices do not rise by as much.

Note that this channel also interacts with the degree of exchange rate pass-through. Under producer currency pricing, the price of goods is sticky in the currency of the producer. Therefore, the changes in import prices are transmitted through the nominal exchange rate, which is captured in the $\mathcal{H}$ matrix. International relative prices are very volatile in the data, and in the presence of GVCs, relative prices affect the firm's marginal cost directly as some inputs are sourced from abroad. The following section will introduce a dynamic, multi-sector version of our model, exploring the importance of international relative price fluctuations for inflation dynamics.

2.2 Multi-sector Production

Now we extend the model to include multiple sectors with an input-output network. This enables us to take predictions of the model to the data and, derive a motivating equation for our empirical analysis. The consumption aggregator now consists of a finite number of sectoral goods indexed by s, which in turn consists of Home and Foreign goods.

$$C = \mathcal{D}(C_1, \dots, C_S) \ \text{ where } \ C_s = \mathcal{C}\left(C_{Hs}, C_{Fs}\right) \tag{11}$$

The utility function remains as in equation (2). There is a representative firm in each sector with the production technology,

$$Y_{Hs}(i) = A_s L_s(i)^{\delta_s} M_s(i)^{1-\delta_s} \ , \tag{12}$$

where intermediate inputs in sector s is,

$$M_s(i) = \left[\sum_{s'=1}^{S} \chi_{ss'}^{\frac{1}{\phi_s}} \left(M_{ss't}(i)\right)^{\frac{\phi_s - 1}{\phi_s}} \right]^{\frac{\phi_s}{\phi_s - 1}} \ , \tag{13}$$

and each intermediate input can come from Home or Foreign sources,

$$M_{ss'}(i) = \left[\mu_{ss'}^{\frac{1}{\phi_{ss'}}} \left(M_{Hss'}(i)\right)^{\frac{\phi_{ss'}-1}{\phi_{ss'}}} + (1-\mu_{ss'})^{\frac{1}{\phi_{ss'}}} \left(M_{Fss'}(i)\right)^{\frac{\phi_{ss'}-1}{\phi_{ss'}}} \right]^{\frac{\phi_{ss'}}{\phi_{ss'}-1}} \ . \tag{14}$$

Sectors may differ across several dimensions including the labor share, its position in the input-output network, the home-bias for intermediate inputs and the elasticity of substitution. Let lowercase variables denotes log deviation from steady state. The linearized sectoral marginal costs in Home can be expressed as,

$$\mathbf{mc} = \delta \cdot w + \Omega^H \mathbf{p}^H + \Omega^F \mathbf{p}^F - \log \mathbf{A} \ , \tag{15}$$

where δ is a vector of labor shares and Ω^H and Ω^F are matrices of steady-state home and imported intermediate goods shares respectively. Formally, $\Omega^H = \{\omega^H_{ss'}\}$ and $\Omega^F = \{\omega^F_{ss'}\}$ where

$$\omega^H_{ss'} \equiv \frac{P^H_{ss'} M^H_{ss'}}{\mathrm{MC}_s Y_s} = \frac{(1-\delta_s)\left(\frac{\chi_{ss'} (\bar{P}^M_{ss'})^{1-\phi_s}}{\sum_{k=1}^{S} \chi_{sk} (\bar{P}^M_{sk})^{1-\phi_s}} \right)}{1 + \left(\frac{1-\mu_{ss'}}{\mu_{ss'}} \right) \left(\bar{P}^{\mathrm{TOT}}_{ss'} \right)^{\phi_{ss'}-1}} \ , \tag{16}$$

and

$$\omega^F_{ss'} \equiv \frac{P^F_{ss'} M^F_{ss'}}{\mathrm{MC}_s Y_s} = \frac{(1-\delta_s)\left(\frac{\chi_{ss'} (\bar{P}^M_{ss'})^{1-\phi_s}}{\sum_{k=1}^{S} \chi_{sk} (\bar{P}^M_{sk})^{1-\phi_s}} \right)}{1 + \left(\frac{\mu_{ss'}}{1-\mu_{ss'}} \right) \left(\bar{P}^{\mathrm{TOT}}_{ss'} \right)^{1-\phi_{ss'}}} \ . \tag{17}$$

The terms-of-trade term extends sector-by-sector definition $\bar{P}^{\mathrm{TOT}}_{ss'} = \frac{\bar{P}^H_{ss'}}{\bar{P}^F_{ss'}}$. The expression of the Foreign counterparts, Ω^{H*}, Ω^{F*} are symmetric. Note that the Global Input-Output matrix in the previous section is now a function of each ot these matrices. That is $\Omega = \mathcal{F}\left(\Omega^H, \Omega^F, \Omega^{H*}, \Omega^{F*}\right)$. We are now ready to derive the sectoral (PPI) Phillips curve.

Proposition 3. *The sectoral (PPI) Phillips curve is*

$$\pi^H = (I - \Delta\Omega^H - \Delta\delta(\boldsymbol{\alpha}^H)^T)^{-1}\Delta\left\{\delta(\gamma+\varphi)\widetilde{y} + (\Omega^F + \delta(\boldsymbol{\alpha}^F)^T)\pi^F + (\delta\boldsymbol{\lambda}^T - I)\log\mathbf{A}\right\} \tag{18}$$

Proof. We start with the primal form of the Phillips curve,

$$\pi^H = \Delta(\mathbf{mc} - \mathbf{p}^H_{-1}) \ .$$

Combining the expression for Home inflation and marginal cost yields

$$(I - \Delta\Omega^H)\ \pi^H = \Delta\ (\delta\cdot w + \Omega^F\pi^F - (I-\Omega^H)\mathbf{p}^H_{-1} + \Omega^F\mathbf{p}^F_{-1} - \log\mathbf{A})\ .$$

Get real wages to appear on the right-hand side

$$\begin{aligned}(I - \Delta\Omega^H - \Delta\delta(\boldsymbol{\alpha}^H)^T)\pi^H = \Delta\Big\{&\delta(w - (\boldsymbol{\alpha}^H)^T\mathbf{p}^H - (\boldsymbol{\alpha}^F)^T\mathbf{p}^F) + (\Omega^F + \delta(\boldsymbol{\alpha}^F)^T)\pi^F \\ &- \log\mathbf{A} - (I - \Omega^H - \delta(\boldsymbol{\alpha}^F)^T)\mathbf{p}^H_{-1} + (\Omega^F + \delta(\boldsymbol{\alpha}^F)^T)\mathbf{p}^F_{-1}\Big\}\end{aligned}$$

Following Rubbo (2020), real wages and the output gap is related through the following equation,

$$w - (\boldsymbol{\alpha}^H)^T\mathbf{p}^H - (\boldsymbol{\alpha}^F)^T\mathbf{p}^F = (\gamma+\varphi)\widetilde{y} + \boldsymbol{\lambda}^T\log\mathbf{A}\ . \tag{19}$$

Combining the previous two equations results in,

$$\begin{aligned}\pi^H = (I - \Delta\Omega^H - \Delta\delta(\boldsymbol{\alpha}^H)^T)^{-1}\\ \Delta\Big\{&\delta(\gamma+\varphi)\widetilde{y} + (\Omega^F + \delta(\boldsymbol{\alpha}^F)^T)\pi^F + (\delta\boldsymbol{\lambda}^T - I)\log\mathbf{A} \\ &- (I - \Omega^H - \delta(\boldsymbol{\alpha}^F)^T)\mathbf{p}^H_{-1} + (\Omega^F + \delta(\boldsymbol{\alpha}^F)^T)\mathbf{p}^F_{-1}\Big\}\end{aligned}$$

As we assume that the model is initially at the steady-state, then $\mathbf{p}^H_{-1} = \mathbf{p}^F_{-1} = 0$ and the last two terms drop out. □

Equation (18) is the motivating equation for our empirical framework. It predicts how the imported intermediate inputs affects sectoral inflation. First, any factor that is not Home labor will be entered separately from the 'slope' of the Phillips curve. Second, the main addition is foreign inflation, which appears on the right-hand side, multiplied by the matrix of imported input shares and the final consumption shares, multiplied by the labor share. Third, the factor costs as a share of total costs are constant as we are considering the effect of small shocks to first order.

Import prices. We can consider the pass-through of nominal exchange rates to imported sectoral goods,

$$\mathbf{p}^F = \mathbf{p}^{F*} + \boldsymbol{\nu}\cdot e\ , \tag{20}$$

where $\boldsymbol{\nu}$ denotes a vector of exchange rate pass-through. The case $v_j = 1$ captures full exchange rate pass-through at the sectoral level, i.e. producer currency pricing, while $v_j = 0$ captures local currency pricing. Tariffs or other distortionary trade barriers can be captured as an ad-valorem tax on the import price. This enters linearly as

$$\mathbf{p}^F = \widetilde{\mathbf{p}}^F + \boldsymbol{\tau} \,. \tag{21}$$

where $\boldsymbol{\tau}$ is a vector of sectoral tariffs. Aside from the case where $v_j = 1$, there is a deviation from the law of one price. Similarly, in partial equilibrium, a 'terms-of-trade' shock would enter linearly to the price of Foreign goods, thereby shifting the Phillips curve as opposed to changing the slope.

3 Empirics

We now turn to the data to examine whether the mechanisms highlighted by our model are reflected in the behavior of sectoral inflation. In particular, the model predicts that greater reliance on imported intermediate inputs should weaken the sensitivity of inflation to domestic slack. To test this prediction, we combine quarterly UK sectoral inflation and output data from the Office of National Statistics (ONS) with annual input-output data from the WIOD for 40 industries over 2000Q1-2014Q4.[3] We construct sector-specific measures of GVC integration based on the share of imported intermediates and interact these with sectoral output gaps to assess how imported input reliance affects the inflation-output gap relationship.[4]

3.1 Sectoral Regressions

Guided by the model, we estimate the following reduced-form specification for 2000Q1–2014Q4,

$$\pi_{jt} = \beta_1 \, \widetilde{y}_{jt} + \beta_2 \, \mathrm{IIS}_{j,t-4} + \beta_3 \, \widetilde{y}_{jt} \times \mathrm{IIS}_{j,t-4} + \beta_4 \left(\frac{1}{4} \sum_{k=1}^{4} \pi_{j,t-k} \right) + \delta_j + \delta_t + \varepsilon_{jt} \,, \tag{22}$$

where $\mathrm{IIS}_{j,t-4}$ is defined above as the ratio of imported intermediate goods in total intermediate goods in sector j at time t. The IIS variable is standardized so that coefficients measure the effect of a one-standard-deviation change in imported-input dependence, facilitating interpretation of interactions. Sectoral inflation series π_{jt} are calculated as the four-quarter percentage change in PPI and SPPI, and $\widetilde{y}_{jt}$ is the sectoral output gap.[5] Panel data allow us to control for time-invariant sector-specific factors using sector fixed effects as well as time-varying aggregate factors affecting inflation such as monetary policy (McLeay and Tenreyro (2020)) and aggregate inflation expectations (Ball and Mazumder (2019)) using a time fixed effect.[6] To control for sector-level inflation expectations, we include a moving average of past sectoral inflation.[7]

The coefficient of interest is the interaction term, β_3. A negative interaction term would imply

[3] We can merge trade, price, and output data for 40 out of 56 WIOD sectors with a balanced panel, and they comprise 70% of total UK output.

[4] We also look at whether aggregate trade openness can affect the relationship between the UK's inflation and the output gap. We find supporting evidence that rising trade openness in the UK is associated with a lower correlation between the inflation and the output gap. However, given that the estimations at the aggregate level are subject to identification issues and that our focus is trade in intermediate inputs, we do not report the results in the main text. See, Appendix B for details.

[5] Both sectoral inflation and output series are at a quarterly frequency and $\mathrm{IIS}_{j,t-4}$ is available at the annual frequency. Details about the data sources can be found in Appendix A.

[6] We use *year* fixed effects in our benchmark analysis, however, our results are robust to using *quarterly* fixed effects. Our results are also robust to including sector trends. Results from these estimations are available upon request from the authors.

[7] Inflation and output data are winsorized at the 1st and 99th percentiles, though results are robust to not winsorizing.

Table 1: GVCs and the UK Phillips Curve

	(1) Output Gap Only	(2) Role of GVCs
$\widetilde{y}_{jt}$	0.0430***	0.0426***
	(0.0138)	(0.0128)
$\text{IIS}_{j,t-4}$		-0.103
		(0.399)
$\widetilde{y}_{jt} \times \text{IIS}_{j,t-4}$		-0.00765
		(0.0216)
Average of Lags	0.376***	0.369***
	(0.0429)	(0.0486)
Industry FE	Y	Y
Time FE	Y	Y
No of Obs.	2158	2030
R^2	0.251	0.244

Note: Results are from equation (22). Column (1) uses the equation without IIS_{jt} term. Column (2) estimates the full equation. Driscoll-Kraay standard errors are in parenthesis with a lag of 8.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

that increased GVC integration is associated with reduced responsiveness of inflation to the domestic output gap. Table 1 presents the results from estimating equation (22). Column (1) shows the positive and significant relationship between sectoral inflation and the output gap. This finding suggests that the UK Phillips curve can be precisely estimated using sectoral data. This aligns with the McLeay and Tenreyro (2020) critique, which suggests that successful monetary policy might have led to a lower correlation between inflation and real economic activity by responding to inflation in a timely manner and attenuating its reaction to demand-side shocks at the aggregate level. Exploiting the panel structure in inflation and output gap, and after controlling for aggregate level time-varying trends using time fixed effects, we find a positive and significant Phillips curve coefficient in the UK within our sample period.

Moving to our main argument that increasing input trade might be related to reduced responsiveness of UK's inflation to domestic demand conditions, we present the results from the interaction of the sectoral output gap with the imported intermediate goods share in column (2). The coefficient on this interaction term (row three) is negative, suggesting that GVCs helps explain the cross-sectoral heterogeneity in the inflation output gap relationship. However, the estimate is imprecise and statistically insignificant, indicating that the variation in imported intermediate input shares is too limited to identify the effect of GVCs on the link between inflation and domestic slack.

Source of Inputs. Given the weak role of overall IIS, we next investigate whether the source of imported intermediates matters. Aggregating across all trading partners may mask offsetting effects. Figure 1 compares the change in the share of AEs and EMEs in intermediate inputs used by three types of sectors, services, and capital goods manufacturing, and non-capital goods manufacturing. Notably, the figure displays a widespread rise in integration to the EMEs compared to the stable levels of dependence on AE imports between 2000 and 2014. This integration is particularly striking in sectors which manufacture capital goods, such as 'Computer Electronics', where the share of intermediate goods from EMEs has increased up to sixfold. Decomposing $\text{IIS}_{j,t-4}$ by source region shows that most of the cross-sectoral heterogeneity is driven by inputs from EMEs rather than from AEs or EU partners. Services sectors such as 'Hospitality' experienced an increase in IIS, but less is coming from EMEs. Non-capital goods manufacturing, which we term 'manufacturing', experienced an increase in IIS from EMEs, but less than that of capital goods manufacturing.

To formally differentiate the roles of integration of the UK sectors to different regions, we estimate equation (22) by distinguishing between different source-region in variable $\text{IIS}_{j,t-4}$ as follows,

$$\text{IIS}^{\text{AE}}_{j,t-4} = \frac{\text{Imported Intermediate Goods}^{\text{AE}}_{j,t-4}}{\text{Total Intermediate Goods}_{j,t-4}}, \quad \text{IIS}^{\text{EM}}_{j,t-4} = \frac{\text{Imported Intermediate Goods}^{\text{EM}}_{j,t-4}}{\text{Total Intermediate Goods}_{j,t-4}},$$

Using this specification allows us to assess how the sensitivity of inflation to domestic slack varies with imported-input intensity from different regions. We standardize each variable around its mean prior to estimation, abstracting from differences in scale.

Table 2 presents the results. The previous estimation result from total imported intermediate goods shares is shown in column (1). The estimated coefficients from columns (2), (3), and (4) provide the striking difference in the role of integration to the EU, AEs, and EMEs on the UK Phillips curve, respectively. Column (4) shows that the coefficient of the interaction term is negative and statistically significant, implying a role for imported intermediate goods shares from EMEs. To state differently, we find that increased integration of the UK sectors to the EMEs led to a diminished response of UK inflation to the output gap between 2000 and 2014. On the other hand, columns (2) and (3) suggest that we cannot precisely estimate the role of integration to the EU or AEs on the UK Phillips curve.

To report the economic significance of the results, recall that $\text{IIS}^{EME}_{j,t-4}$ is standardized; thus, the coefficient for the output gap (0.0389) denotes the Phillips curve coefficient for the mean level of integration to the EMEs. The coefficient of the interaction term (-0.0421) implies that one standard deviation increase in the share of imported intermediate goods from EMEs in UK sectors reduces the correlation between the inflation and the output gap to near 0. Furthermore, we apply back-of-the-envelope calculations to understand the importance of rising imported intermediate goods dependence on the EMEs on the value of the UK Phillips curve slope. Using the coefficients from column (4), we find that the Phillips curve coefficient reduced by 64% between 2000 and 2014 due to rising $\text{IIS}^{EME}_{j,t-4}$, after controlling for aggregate time-varying sector-specific time-invariant effects.

Our findings shed new light on how shifts in the UK's participation in GVCs may influence the responsiveness of inflation to domestic economic conditions. Rather than focusing solely on the overall degree of trade integration, we highlight the importance of where UK sectors source their imported intermediates. In particular, distinguishing integration with EMEs from integration with AEs or the EU reveals that the regional composition of imported inputs plays a key role in shaping the inflation-activity relationship. This shows that changes in the direction of GVC integration are central in understanding inflation dynamics.

3.2 Robustness Checks

Including Lags. We first aim to isolate the impact of the increased use of imported intermediates in production on the link between inflation and the output gap. To do so, we use the lags of our GVCs measure in our regressions. Furthermore, we calculate the two- and three-year moving average in IIS^{EM}_{jt} to smooth short-run relative price fluctuations.

Table 2: GVCs and the UK Phillips Curve: The Source Matters

	(1) Total	(2) EU	(3) AEs	(4) EMEs	(5) EMEs vs. AEs
$\widetilde{y}_{jt}$	0.0426***	0.0418***	0.0432***	0.0389***	0.0354***
	(0.0128)	(0.0133)	(0.0137)	(0.0104)	(0.0116)
$\text{IIS}_{j,t-4}$	-0.103				
	(0.399)				
$\widetilde{y}_{jt} \times \text{IIS}_{j,t-4}$	-0.00765				
	(0.0216)				
$\text{IIS}^{\text{EU}}_{j,t-4}$		-0.144			
		(0.541)			
$\widetilde{y}_{jt} \times \text{IIS}^{\text{EU}}_{j,t-4}$		-0.00290			
		(0.0190)			
$\text{IIS}^{\text{AE}}_{j,t-4}$			-0.401		-0.752*
			(0.416)		(0.435)
$\widetilde{y}_{jt} \times \text{IIS}^{\text{AE}}_{j,t-4}$			-0.00127		0.0414*
			(0.0187)		(0.0211)
$\text{IIS}^{\text{EME}}_{j,t-4}$				0.389	0.583***
				(0.234)	(0.196)
$\widetilde{y}_{jt} \times \text{IIS}^{\text{EME}}_{j,t-4}$				-0.0421**	-0.0732***
				(0.0172)	(0.0243)
Industry FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
No of Obs.	2030	2030	2030	2030	2030
R^2	0.244	0.244	0.245	0.249	0.255

Note: Results are from equation (22). Columns (1)-(4) use $\text{IIS}_{j,t-4}$, $\text{IIS}^{EU}_{j,t-4}$, $\text{IIS}^{\text{AE}}_{j,t-4}$, $\text{IIS}^{\text{EME}}_{j,t-4}$, respectively. Column (5) includes both $\text{IIS}^{\text{AE}}_{j,t-4}$ and $\text{IIS}^{\text{EME}}_{j,t-4}$ in the regression. Driscoll-Kraay standard errors are in parenthesis with a lag of 8. * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

Table 3 presents the results with a baseline specification (column (1)), using the lag of our GVC measurement $IIS^{EM}_{j,t-4}$ (column (2)), two-year moving average and three-year moving average. The interaction terms from each column suggest that integration to the EMEs is associated with a flatter Phillips curve in the UK consistently with the theoretical results presented in section 2.

Instrumental Variable Analysis. Following the trade literature, we assess the potential endogeneity problem due to including the $\text{IIS}_{j,t-4}$ variable in equation (22) which may bias the interpretation of its relationship with sectoral inflation and activity. As emphasized by Autor et al. (2013), growth in imports may not only reflect increased competitiveness or productivity in the source country; it can also be driven by rising demand in the importing country. Because stronger domestic demand is typically associated with higher inflation, estimations could suffer from endogeneity. As a result, OLS estimates may be biased and can understate the true effect.

To isolate the exogenous variation in imported intermediate input share, we follow Autor et al. (2013). We estimate the following structural equation and the first stage of the IV specification,

$$\pi_{jt} = \beta_1 \widetilde{y}_{jt} + \beta_2 \text{IIS}_{j,t-4} + \beta_3 \widetilde{y}_{jt} \times \text{IIS}_{j,t-4} + \beta_4 \left(\frac{1}{4} \sum_{k=1}^{4} \pi_{j,t-k} \right) + \delta_j + \delta_t + \epsilon_{jt} \tag{23}$$

Table 3: Further Controls on Medium-term Impacts

	(1) Two-Year Moving Average	(2) Three-Year Moving Average
$\tilde{y}_{jt}$	0.0358*	0.0324
	(0.0206)	(0.0205)
$\overline{\text{IIS}}_{j,t-8\to t-4}$	-0.0617	
	(0.293)	
$\tilde{y}_{jt} \times \overline{\text{IIS}}_{j,t-8\to t-4}$	-0.0363*	
	(0.0204)	
$\overline{\text{IIS}}_{j,t-12\to t-4}$		-0.135
		(0.324)
$\tilde{y}_{jt} \times \overline{\text{IIS}}_{j,t-12,t-4}$		-0.0412*
		(0.0209)
Industry FE	Y	Y
Time FE	Y	Y
No of Obs.	1877	1720
R^2	0.549	0.561

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

$$\text{IIS}_{j,t-4} = \alpha\ \text{IIS}^{Others}_{j,t-4} + \delta_j + \delta_t + \eta_{jt}\ , \tag{24}$$

where we use the imports of 8 other developed countries from EMEs and China separately to calculate $\text{IIS}^{Others}_{j,t-4} = \frac{\text{Imported Intermediate Goods}^{Others}_{j,t-4}}{\text{Total Intermediate Goods}_{j,t-4}}$.[8] Here, the identification assumption is that the import demand shocks at the sector level between the UK and 8 other developed countries are independent.[9] Table 4 shows that the estimated effects of GVC integration with both EMEs (columns 1-2) and China (columns 3-4) are robust to the IV strategy. The coefficients on interaction terms are slightly higher in absolute value and remain statistically significant at 5% level.

4 Dissecting the mechanism: Why EMEs?

Our empirical findings show that GVC integration with EMEs systematically shapes the sectoral inflation-output gap relationship. However, the reduced-form evidence does not speak directly to the underlying mechanisms. In particular, it remains unclear why integration with EMEs, rather than with other advanced economies, drives the patterns we document. Guided by the trends in Figure 1, we highlight two key features of the UK's integration dynamics: (i) a *source* dimension, as EME-sourced intermediates have risen sharply over the sample period, and (ii) a *sectoral* dimension, since this increase has been disproportionately concentrated in capital goods industries. To evaluate whether these features can help understand our empirical results, we develop a quantitative two-country model of GVCs and conduct counterfactual exercises that isolate the role of sectoral and source specific changes in GVCs in shaping the inflation-output relationship.

[8]These include Australia, Denmark, Finland, Germany, Japan, Spain, Switzerland, United States. The correlation between the instrument and the endogenous regressor is 0.85.

[9]The results are robust to using G7 countries or only the U.S. for instrumenting the UK's imports.

Table 4: Instrumental Variable Analysis

	(1) OLS	(2) IV	(3) OLS	(4) IV
$\widetilde{y}_{jt}$	0.0389***	0.0408***	0.0397***	0.0383***
	(0.0104)	(0.0118)	(0.0103)	(0.0102)
$\text{IIS}^{\text{EM}}_{j,t-4}$	0.389	0.379		
	(0.234)	(0.708)		
$\widetilde{y}_{jt} \times \text{IIS}^{\text{EM}}_{j,t-4}$	-0.0421**	-0.0456***		
	(0.0172)	(0.0161)		
$\text{IIS}^{\text{CH}}_{j,t-4}$			0.356**	0.540**
			(0.146)	(0.242)
$\widetilde{y}_{jt} \times \text{IIS}^{\text{CH}}_{jt-4}$			-0.0464***	-0.0470***
			(0.0141)	(0.0128)
First-stage Fstat		1048.6		520.7
Industry FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
No of Obs.	2030	2030	2030	2030
R^2	0.249	0.248	0.250	0.251

Standard errors in parentheses

Driscoll-Kraay standard errors are in parentheses with a lag of 8

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.1 A New-Keynesian Model with Global Value Chains

In this section, we introduce a two-country, multi-sector New Keynesian model with sectoral linkages. The two countries, Home (H) and Foreign (F), are populated by a continuum of infinitely lived households with a fraction of n and $1-n$ of the total world population, respectively. Relative to the static model in Section 2, this model extends the setup to allow for dynamics and general equilibrium.[10]

In each country, there is a continuum of firms indexed by $i \in [0, 1]$ and each firm belongs to a sector, $s \in 1, ..., S$. Firms produce differentiated products which can be sold domestically or exported for consumption and production. Our model incorporates GVCs through trade in intermediate inputs. In each sector, monopolistically competitive firms produce their output using labor and intermediate goods as inputs. Each period, producers choose how much intermediate input they want to buy from each sector and then they decide whether to buy home or foreign-produced intermediates. Similarly, we assume that aggregate consumption is a composite of sectoral consumption goods and each of these goods is a CES aggregate of home and foreign-produced goods. Thus, there is trade in final goods as well. Changes in global value chains are captured through transitory changes in tariffs for imported intermediate inputs and final goods. For brevity, we present the equations only for the Home economy. The Foreign economy has a symmetric setup.

4.2 Model Setup

Preferences. Each period households optimally allocate their total expenditure across sectoral goods. The final consumption basket is a CES aggregate of finitely many sectoral goods $s \in \{1, 2, \ldots, S\}$

[10]The modeling choices are standard. For instance Comin and Johnson (2020) presents a similar small open economy model with Rotemberg price adjustments instead of Calvo. We differ from them by presenting a two-country structure that accounts for the endogenous changes in terms of trade.

in each country,

$$C_t = \left[\sum_{s=1}^{S} \eta_s^{\frac{1}{\phi_C}} \left(C_{st} \right)^{\frac{\phi_C - 1}{\phi_C}} \right]^{\frac{\phi_C}{\theta_C - 1}} , \tag{25}$$

where ϕ_C is elasticity of substitution between sectoral consumption goods and η_s is the share of sector s in total consumption with $\sum_{s=1}^{S} \eta_s = 1$. Sectoral goods themselves are also CES aggregates of Home (C_{Hst}) and Foreign (C_{Fst}) consumption goods such that,

$$C_{st} = \left[\alpha_s^{\frac{1}{\phi_{Cs}}} \left(C_{Hst} \right)^{\frac{\phi_{Cs} - 1}{\phi_{Cs}}} + (1 - \alpha_s)^{\frac{1}{\phi_{Cs}}} \left(C_{Fst} \right)^{\frac{\phi_{Cs} - 1}{\phi_{Cs}}} \right]^{\frac{\phi_{Cs}}{\phi_{Cs} - 1}} , \tag{26}$$

where α_s represents the share of home-produced goods in sectoral consumption and ϕ_{Cs} is the elasticity of substitution between home and foreign-produced consumption goods which is allowed to be different across sectors. As in the static model, the share of imported goods in each sector is a function of relative country size, $1 - n$, and the degree of openness in final demand, v_{Cs}: $1 - \alpha_s = (1 - n)\, v_{Cs}$. When $\alpha_s > 0.5$, there is home bias in preferences in a given sector. Household expenditure minimization yields the following optimal demand for sectoral goods,

$$C_{st} = \eta_s \left(\frac{P_{st}}{P_t} \right)^{-\phi_C} C_t, \tag{27}$$

where the aggregate price index is $P_t = \left[\sum_{s=1}^{S} \eta_s P_{st}^{1-\phi_C} \right]^{\frac{1}{1-\phi_C}}$. Sectoral consumption is further allocated between home and foreign goods according to,

$$C_{Hst} = \alpha_s \left(\frac{P_{Hst}}{P_{st}} \right)^{-\phi_{Cs}} C_{st} \text{ and } C_{Fst} = (1 - \alpha_s) \left(\frac{(1 + \tau_{st}) P_{Fst}}{P_{st}} \right)^{-\phi_{Cs}} C_{st} , \tag{28}$$

where the sectoral price index is $P_{st} = \left[\alpha_s P_{Hst}^{1-\phi_{Cs}} + (1 - \alpha_s)((1 + \tau_{st}) P_{Fst})^{1-\phi_{Cs}} \right]^{\frac{1}{1-\phi_{Cs}}}$. Imported final goods are subject to tariffs τ_{st}, which creates a deviation from the law of one price. Households derive utility from consumption and disutility from supplying labor. The lifetime utility function of the representative household is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1 - \sigma} - \Xi \frac{L_t^{1+\varphi}}{1 + \varphi} \right\}, \tag{29}$$

where β is the discount factor and Ξ is a preference parameter. Households finance expenditure on consumption goods through labor income and profits from the ownership of firms. We assume that the international asset markets are complete in the sense that households trade state-contingent securities that are denominated in the home currency to buy consumption goods. Additionally, we assume that only bonds that are issued by Home are traded internationally. The period budget constraint of the home household is

$$P_t C_t + \sum Q_{t,t+1} B_{Ht,t+1} \leq B_{Ht-1,t} + W_t L_t + D_t + T_t , \tag{30}$$

where P_t is the CPI, W_t is the nominal wage and D_t are firm profits. $B_{Ht,t+1}$ denotes the home households holding of nominal state-contingent internationally traded bonds which deliver one unit of home currency in period $t+1$ if a particular state occurs. $Q_{t,t+1}$ is the price of such bond at time t. The government provides lump-sum transfers T_t to the household, which consists of rebating the revenues from tariffs. Solving the household's problem yields a standard Euler equation, intra-temporal equation and Backus-Smith condition.

Firms. The supply side of the economy consists of perfectly competitive sectoral producers at the retail level and monopolistically competitive firms at the wholesale level. Infinitely many competitive firms aggregate firm-level domestic varieties $Y_{Hst}(i)$ into sectoral goods Y_{Hst} using the following technology,

$$Y_{Hst} = \left[\int_0^1 Y_{Hst}^{\frac{\epsilon_s}{\epsilon_s - 1}}(i) di \right]^{\frac{\epsilon_s - 1}{\epsilon_s}} , \tag{31}$$

where ϵ_s is the elasticity of substitution between varieties within a sector. The solution to this aggregation problem implies the following demand for varieties,

$$Y_{Hst}(i) = \left(\frac{P_{Hst}(i)}{P_{Hst}} \right)^{-\epsilon_s} Y_{Hst} \, . \tag{32}$$

Firms use labor and intermediate inputs to produce a unit of output. The production function is given by,

$$Y_{Hst}(i) = A_t A_{st} L_{st}(i)^{\delta_s} M_{st}(i)^{1-\delta_s} \, , \tag{33}$$

where L_{st} denotes firm i's labor demand and δ_s denotes the share of labor in production. Aggregate and sectoral productivity assumed to follow an AR(1) process and are represented by A_t and A_{st} respectively.[11] Each firm i uses intermediate good, $M_{st}(i)$, which is a CES aggregate of sectoral goods,

$$M_{st}(i) = \left[\sum_{s'=1}^{S} \chi_{ss'}^{\frac{1}{\phi_M}} \left(M_{ss't}(i) \right)^{\frac{\phi_M - 1}{\phi_M}} \right]^{\frac{\phi_M}{\phi_M - 1}} , \tag{36}$$

where $M_{ss't}$ is the intermediate good demand of sector s from sector s' at time t, and $\chi_{ss'}$ is the share of sector s' in total intermediate good expenditure of sector s with $\sum_{s'=1}^{S} \chi_{ss'} = 1$. The elasticity of substitution across sectoral intermediate goods is denoted by ϕ_M. Firms' sectoral input demand is a CES aggregate of domestic and foreign intermediate goods as in the consumption case,

$$M_{ss't}(i) = \left[\mu_{ss'}^{\frac{1}{\phi_{Ms}}} \left(M_{Hss't}(i) \right)^{\frac{\phi_{Ms} - 1}{\phi_{Ms}}} + (1 - \mu_{ss'})^{\frac{1}{\phi_{Ms}}} \left(M_{Fss't}(i) \right)^{\frac{\phi_{Ms} - 1}{\phi_{Ms}}} \right]^{\frac{\phi_{Ms}}{\phi_{Ms} - 1}} , \tag{37}$$

where $M_{Hss't}(i)$ and $M_{Fss't}(i)$ denote domestic and foreign intermediate good demand of sector s from sector s' at time t, respectively. There exists sectoral home bias at the intermediate level denoted

[11] The law of motion for aggregate and sectoral productivity are

$$\log A_t = (1 - \rho_A) \log \overline{A} + \rho_A \log A_{t-1} + \varepsilon_{At}, \tag{34}$$
$$\log A_{st} = (1 - \rho_{As}) \log \overline{A_s} + \rho_{As} \log A_{st-1} + \varepsilon_{Ast}, \tag{35}$$

where ε_{At} and $\varepsilon_{A_s t}$ are i.i.d. innovations.

by $\mu_{ss'}$, and ϕ_{Ms} denotes the elasticity of substitution between home and foreign-produced intermediate goods which is allowed to be different across sectors. Similar to consumption preference structure, we assume that the share of imported intermediate goods is a function of relative country size and the degree of openness in intermediate goods in a sector.

Every period, firms choose the labor and intermediate inputs to minimize their costs. Optimal input demands are,

$$L_{st} = \delta_s \left(\frac{MC_{st}}{W_t} \right) Y_{Hst} \text{ and } M_{st} = (1 - \delta_s) \left(\frac{MC_{st}}{P^M_{st}} \right) Y_{Hst},$$

where MC_{st} is sectoral marginal cost (will be defined below) and P^M_{st} is the intermediate input price index for sector s. Firms also optimally choose sectoral intermediate goods as,

$$M_{ss't} = \chi_{ss'} \left(\frac{P^M_{ss't}}{P^M_{st}} \right)^{-\phi_M} M_{st},$$

where intermediate input price index is $P^M_{st} = \left[\sum_{s'=1}^{S} \chi_{ss'} \left(P^M_{ss't} \right)^{1-\phi_M} \right]^{\frac{1}{1-\phi_M}}$, and the demand for home and foreign sectoral inputs is given by,

$$M_{Hss't} = \mu_{ss'} \left(\frac{P_{Hs't}}{P^M_{ss't}} \right)^{-\phi_{Ms}} M_{ss't} \text{ and } M_{Fss't} = (1 - \mu_{ss'}) \left(\frac{(1 + \tau_{s't}) P_{Fs't}}{P^M_{ss't}} \right)^{-\phi_{Ms}} M_{ss't},$$

where sectoral intermediates price index is a weighted average of home and foreign sectoral output prices $P^M_{ss't} = \left[\mu_{ss'} P^{1-\phi_{Ms}}_{Hs't} + (1 - \mu_{ss'})((1 + \tau_{s't}) P_{Fs't})^{1-\phi_{Ms}} \right]^{\frac{1}{1-\phi_{Ms}}}$. Note that imported intermediate goods are also subject to tariffs. Using firms' demand for factors of production, we can derive the sectoral nominal marginal cost,

$$MC_{st} = \frac{1}{A_t A_{st}} \left(\frac{W_t}{\delta_s} \right)^{\delta_s} \left(\frac{P^M_{st}}{1 - \delta_s} \right)^{1-\delta_s}. \tag{38}$$

Note that sectoral linkages through input-output relationships at the intermediate goods level imply a sectoral marginal cost that depends on other sectors' output prices.

Pricing. We assume that firms are subject to Calvo-type price rigidities such that a firm can update its price with a probability of $1 - \theta_s$, where θ_s denotes the sector-specific price stickiness. Wholesalers can re-set its price, maximizes the present discounted future value of profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_s)^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \{ P_{Hst}(i) Y_{Hst}(i) - MC_{st}(i) Y_{Hst}(i) \} , \tag{39}$$

subject to demand function (32). We assume that the law of one price holds such that the pre-tariff price of foreign goods in the units of home currency is $P_{Fst} = \mathcal{E}_t P^*_{Fst}$ and the pre-tariff price of home goods in the units of Foreign currency is $P^*_{Hst} = \frac{P_{Hst}}{\mathcal{E}_t}$.[12]

[12]We acknowledge that imperfect exchange rate pass-through can be important to understand the fluctuations in international relative prices as explored by Devereux and Engel (2002). Nevertheless, we follow Galí and Monacelli (2005) and

Government. The government rebates tax revenues lump-sum to the household and has a balanced budget in every period. Tax revenues consists of tariffs on imported intermediate and final goods such that,

$$T_t = \tau_{st} P_{Fst} \sum_{s=1}^{S} \left(C_{Fst} + \sum_{s'=1}^{S} M_{Fs'st} \right) . \tag{40}$$

Market Clearing. Labor is perfectly mobile across sectors but not across countries. Therefore, the labor market clearing conditions is,

$$L_t = \sum_{s=1}^{S} L_{st} . \tag{41}$$

Sectoral goods can be used domestically for consumption and for further production as intermediate inputs or they can be exported. Exports can be consumed by foreign consumers or used by foreign firms as inputs. Thus, we can write the market clearing conditions for Home and Foreign goods as,[13]

$$Y_{Hst} = C_{Hst} + \sum_{s'=1}^{S} M_{Hs'st} + \left(\frac{1-n}{n} \right) \left(C^*_{Hst} + \sum_{s'=1}^{S} M^*_{Hs'st} \right) , \tag{44}$$

$$Y_{Fst} = C^*_{Fst} + \sum_{s'=1}^{S} M^*_{Fs'st} + \left(\frac{n}{1-n} \right) \left(C_{Fst} + \sum_{s'=1}^{S} M_{Fs'st} \right) . \tag{45}$$

The resource constraint states that goods produced in each country must be consumed as final goods, used in intermediate production, or lost due to iceberg costs.

Combining the household and government's budget constraints (30), (40), along with the Home goods market clearing condition (44), yields the Home country's budget constraint,

$$\sum Q_{t,t+1} B_{Ht,t+1} - B_{Ht-1,t} = P_{Ht} \left(Y_t - \sum_{s=1}^{S} \left[C_{Hst} + \sum_{s'=1}^{S} M_{Hs'st} \right] \right) - \left(\frac{1-n}{n} \right) P_{Ft} \left(\sum_{s=1}^{S} \left[C_{Fst} + \sum_{s'=1}^{S} M_{Fs'st} \right] \right) ,$$

where the left-hand-side is the (nominal) change in the state-contingent internationally traded assets and the right-hand-side captures the trade balance.

Monetary policy. Monetary policy authority sets the nominal interest rate following an inertial Taylor rule which targets CPI inflation,

$$\frac{I_t}{\bar{I}} = \left(\frac{I_{t-1}}{\bar{I}} \right)^{\Gamma_i} \left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\Gamma_\pi (1-\Gamma_i)} \exp(\varepsilon_{mt}), \tag{46}$$

focus on producer currency pricing to single out the mechanism at play.

[13] Alternatively, one can model the integration of GVCs through 'lost' resources akin to an iceberg trade cost. This captures more technological features of trade. Whereas tariff revenues are rebated to the household, iceberg trade costs leads to a loss in resources for the country as a whole. The market clearing conditions would change to

$$Y_{Hst} = C_{Hst} + \sum_{s'=1}^{S} M_{Hs'st} + \left(\frac{1-n}{n} \right) (1 + \tau^*_{st}) \left(C^*_{Hst} + \sum_{s'=1}^{S} M^*_{Hs'st} \right) , \tag{42}$$

$$Y_{Fst} = C^*_{Fst} + \sum_{s'=1}^{S} M^*_{Fs'st} + \left(\frac{n}{1-n} \right) (1 + \tau_{st}) \left(C_{Fst} + \sum_{s'=1}^{S} M_{Fs'st} \right) . \tag{43}$$

The main difference is with tariffs is the income effect.

where ε_{mt} is a monetary policy shock.

4.3 Calibration

The model is calibrated such that Home represents advanced economies and Foreign represents emerging market economies. The economy consists of three sectors: capital goods, manufacturing and services in this order.[14] This three sector structure keeps the model tractable while capturing the key patterns in sectoral size, openness, and input-output linkages observed in the data.

Preferences are standard: the discount factor is set to $\beta = 0.99$, the coefficient of relative risk aversion to $\sigma = 1$, and the inverse Frisch elasticity to $\varphi = 2$. The disutility of hours parameter is calibrated to match steady-state hours worked of 0.33. The elasticities of substitution across sectors in final and intermediate demand are set to $\phi_C = \phi_M = 0.9$, and the elasticities between Home and Foreign goods within each sector are set to $\phi_{Cs} = \phi_{Ms} = 2$. Monetary policy is described by a Taylor rule with coefficient on inflation $\Gamma_\pi = 2$ and interest-rate smoothing $\Gamma_i = 0.7$.

Sectoral consumption shares (η_s), home bias parameters in final consumption demand (α_s), and labor shares (δ_s) are taken from WIOD. These imply that services is the largest in terms of consumption share and most labor-intensive sector; manufacturing is of intermediate size and has the lowest labor share; and capital goods is the smallest sector with a moderate labor share.

The input-output matrix $(\chi_{i,j})$, also calculated from WIOD, implying strong cross-sector linkages. Capital goods uses relatively more inputs from other sectors, while services relies predominantly on its own inputs, consistent with its downstream position. Manufacturing has an intermediate position. The home bias parameters for intermediate inputs $(\mu_{i,j})$ imply that capital goods is the most import-intensive sector, whereas services sources most of its intermediates domestically.

Sector-specific Calvo parameters are set to $\theta_s = 0.75$, and the elasticity of substitution across varieties within each sector is set to $\epsilon_s = 6$, implying 20% price mark-up. Country size is fixed at $n = 0.5$. The full set of parameter values is reported in Table 5.

4.4 Dissecting the channels: GVCs in the Model

How does GVC integration affect the link between inflation and domestic slack? In our empirical analysis, we measure GVCs as the imported intermediate inputs as a share of total intermediate inputs:

$$GVC_{st}(\tau) = \frac{n}{1-n}\frac{\sum_{s'=1}^{S} P_{Fs't} M_{Fss't}}{P_{st}^M M_{st}} = \frac{\sum_{s'=1}^{S} P_{Fs't}(\tau_{st})(1-\mu_{ss'})\left(\frac{P_{Fs't}}{P_{ss't}^M}\right)^{-\phi_{Ms}} \chi_{ss'}\left(\frac{P_{ss't}^M}{P_{st}^M}\right)^{-\phi_M} M_{st}}{P_{st}^M M_{st}},$$

In the model, GVC integration is thus a function of both structural parameters and relative prices. The parameter $\mu_{ss'}$ governs the share of foreign intermediates in sector s, so a lower μ corresponds to deeper GVC integration. But the measure is not purely technological: it also responds to international relative prices, and therefore to terms-of-trade movements, whenever the elasticities of substitution ϕ_{Ms} (between home and foreign intermediates) and ϕ_M (across sectoral intermediates) differ from 1.

To see this clearly, consider the benchmark Cobb-Douglas case where $\phi_{Ms} = \phi_M = 1$. Then GVC

[14] Sector definitions are provided in Table A1.

Table 5: Baseline Calibration

	Parameter	Value
Households		
σ	Coefficient of relative risk aversion	1
Ξ	Relative disutility of hours	1
φ	Inverse Frisch elasticity	2
β	Discount factor	0.99
Firms		
ϕ_C	Elasticity of substitution across sectors in final consumption	0.9
$(\phi_{Cs})_{s=1,2,3}$	Elasticity of substitution between Home and Foreign goods in final consumption	[2, 2, 2]
$(\eta_s)_{s=1,2,3}$	Sectoral expenditure shares in final consumption	[0.05, 0.22, 0.73]
$(\alpha_s)_{s=1,2,3}$	Home bias in final goods	[0.33, 0.54, 0.94]
$(\delta_s)_{s=1,2,3}$	Labor shares	[0.33, 0.31, 0.58]
ϕ_M	Elasticity of substitution across sectors in intermediate inputs	0.9
$(\phi_{Ms})_{s=1,2,3}$	Elasticity of substitution between Home and Foreign intermediates	[2, 2, 2]
$(\chi_{ss'})_{s,s'}$	Input-output matrix	$\begin{pmatrix} 0.39 & 0.04 & 0.04 \\ 0.27 & 0.51 & 0.16 \\ 0.34 & 0.45 & 0.79 \end{pmatrix}$
$(\mu_{ss'})_{s,s'}$	Home bias in intermediate inputs	$\begin{pmatrix} 0.24 & 0.35 & 0.30 \\ 0.70 & 0.60 & 0.69 \\ 0.90 & 0.91 & 0.91 \end{pmatrix}$
$(\theta_s)_{s=1,2,3}$	Price stickiness	[0.75, 0.75, 0.75]
$(\epsilon_s)_{s=1,2,3}$	Elasticity of substitution across varieties	[6, 6, 6]
Other Parameters		
n	Country size	0.5
Γ_π	Taylor rule coefficient on inflation	2
Γ_i	Interest-rate smoothing	0.7

integration simplifies to:

$$\frac{\sum_{s'=1}^{S} P_{Fs't} M_{Fss't}}{P_{st}^{M} M_{st}} = \sum_{s'=1}^{S} (1 - \mu_{ss'})\, \chi_{ss'}.$$

so the empirical measure captures only openness in production, not relative-price movements. In practice, however, empirical estimates of the elasticity of substitution between home and foreign goods are rarely close to one (see, for e.g Feenstra (1994)), implying that measured GVC intensity varies with the terms-of-trade as well as with the underlying import share. This implies that our empirical measure of GVC integration reflects not only underlying openness in production but also fluctuations in relative prices.

Tariff shocks. To disentangle these forces and illuminate the mechanisms behind our empirical findings, we introduce a sector-specific tariff shock, τ_{st}, which raises the cost of foreign intermediates and mechanically reduces measured GVC integration. Figure 2 reports impulse responses to a 1% increase in sectoral tariffs. The first row presents the responses to a tariff shock in the capital-goods sector, while the second and third rows show the corresponding responses for manufacturing and services. By lowering the foreign-input share, the tariff shock captures a sector-specific GVC fragmentation (or de-integration) exercise.

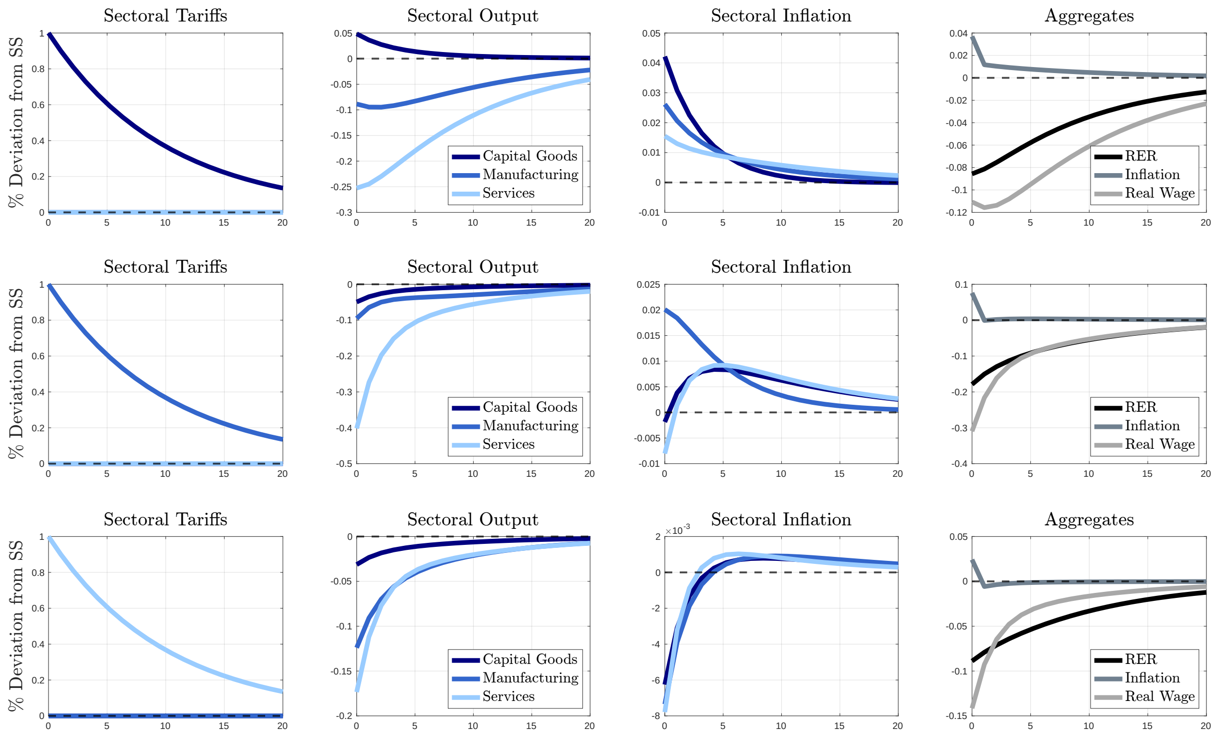
Across all cases, the aggregate responses are qualitatively similar: sectoral tariff increases raise aggregate inflation, depress output and real wages, and generate a real appreciation of Home relative to Foreign. However, transmission to sectoral variables differs substantially depending on the sector affected and the input-output linkages. As shown in the first row of Figure 2, a tariff shock to the capital-goods sector raises its own inflation and output, while reducing output in other sectors and increasing their inflation. In contrast, shocks to manufacturing (second row) or services sectors (third row) produce different patterns: manufacturing shocks generate trade-offs across sectors, lowering output and raising inflation, whereas services shocks act more like a negative demand shock, reducing both output and inflation broadly.

Counterfactual exercises. To shed light on the mechanisms behind the heterogeneous sectoral responses, we conduct a set of counterfactuals that isolate the contribution of input-output linkages. In the model, firms' optimal production choices, labor demand and the allocation of intermediate input expenditure across sectors and across domestic versus foreign sources, are governed by three parameter blocks: sector-specific labor shares (δ_s), the input-output matrix ($\chi_{ss'}$), and the foreign share in intermediate inputs ($1 - \mu_{ss'}$). While labor shares differ across sectors, varying δ_s mainly rescales the effective production technology and therefore has limited implications for the cross-sectoral propagation of shocks. By contrast, input-output linkages and foreign input dependence vary substantially across sectors, precisely along the dimensions emphasized in the empirical analysis. We therefore focus our counterfactual exercises on $\chi_{ss'}$ and $\mu_{ss'}$ to isolate how differences in production network structure and foreign input share shape the transmission of sectoral GVC shocks.

In Figure 3, we shut down firms' ability to source intermediates from other sectors and instead impose a "roundabout production" structure in which each sector can only use its own intermediates. This isolates the role of input-output linkages in the transmission of sector-specific GVCs shocks.

Once inter-sectoral sourcing is eliminated, all GVC shocks generate a decline in output and infla-

Figure 2: Impulse Response Functions to Sectoral Tariffs



Notes: This figure shows the IRFs to a sectoral tariff shock. Row 1 shows the IRFs to a tariff shock in the capital-goods sectors. Row 2 shows the IRFs to a tariff shock in the manufacturing sector. Row 3 shows the IRFs to a tariff shock in the services sector.

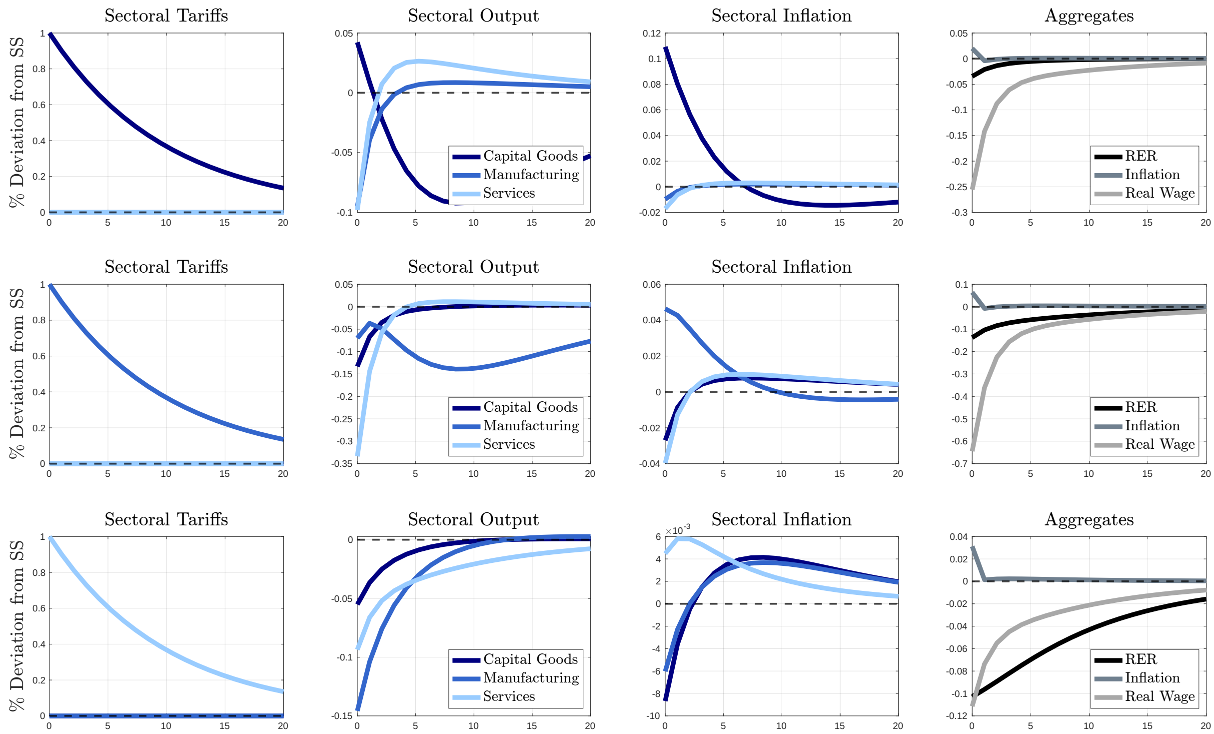
tion in the non-shocked sectors. This pattern reflects the negative income effect: de-integration raises the cost of imported intermediates, lowering real wages at Home and depressing demand in other sectors. In the benchmark economy, however, sectoral linkages re-route these shocks in different ways. Capital goods de-integration behaves like a *positive* demand disturbance for that sector, while services de-integration behaves like a *negative* demand shock. When input-output linkages are shut down, both instead transmit like a negative cost-push shock: for capital goods, output initially rises slightly but then falls, while for services the demand-like response disappears entirely; its inflation rises and output falls.

These results show that the fact that sectoral inflation and output move in the same direction in response to de-integration shocks in the capital goods and services sectors is largely driven by the production network structure. The underlying disturbance is a sector-specific tariff shock that increases production costs in the affected sector. When input-output linkages are shut down, this cost increase translates directly into higher prices and lower output in that sector. Without the ability to substitute toward intermediates from other sectors, marginal costs rise more sharply because the shock becomes more localized rather than being dispersed across the network.

Intuitively, because sectoral linkages flatten the Phillips curve of the shocked sector, part of the cost increase can be passed through to or absorbed by upstream and downstream industries. This dampens the pure cost-push component and contributes to the more demand-like dynamics observed in the benchmark model.

To further disentangle the forces behind the heterogeneous transmission of GVC shocks, we next examine the role of home bias in intermediate input sourcing. While the capital-goods sector is

Figure 3: Counterfactual with no Input-Output Network



Notes: This figure shows the IRFs to a sectoral tariff shock. Row 1 shows the IRFs to a tariff shock in the capital-goods sectors. Row 2 shows the IRFs to a tariff shock in the manufacturing sector. Row 3 shows the IRFs to a tariff shock in the services sector.

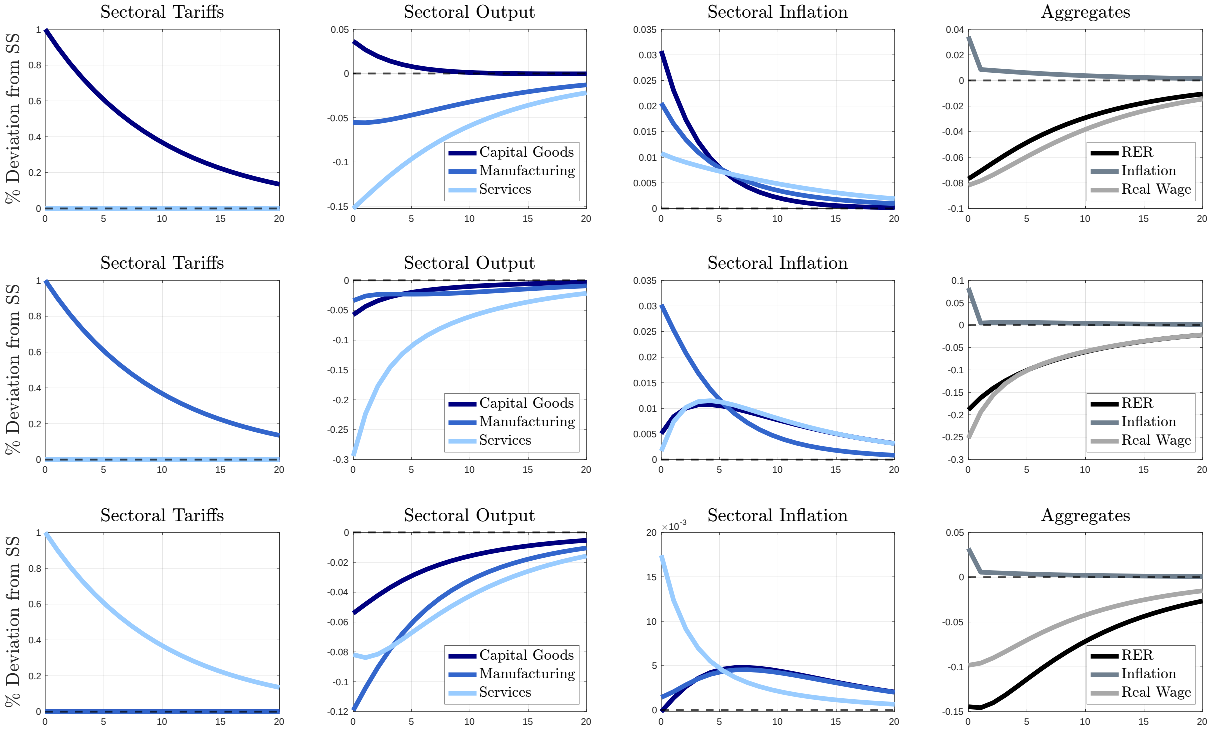
highly open in the benchmark calibration, the services sector is unsurprisingly closed, relying overwhelmingly on domestic inputs. This asymmetry turns out to be central for understanding why the benchmark model generates a *negative* demand-type response for services in response to a sectoral tariff shock.

Figure 4 shows the counterfactual in which we eliminate home bias in intermediate inputs, domestic and foreign inputs now enter symmetrically, while preserving the baseline input-output structure across sectors. Under this experiment, the GVC shock continues to increase both its inflation and output in the capital-goods sector. By contrast, the services sector no longer displays the fall in inflation seen in the benchmark. Instead, its inflation rises while output falls.

The mechanism is straightforward. In the benchmark economy, the high home bias in services insulates the sector from rising foreign input prices. Because imported intermediates represent only a small fraction of its total costs, the tariff shock has a limited impact on the service sector's marginal cost, while demand falls due to negative income effect and generate a decline in both services inflation and output. When we remove this asymmetry and make services more open, the shock feeds directly and more strongly into marginal cost. As a result, the cost push component becomes dominant.

Why do output and inflation in the capital-goods sector increase in response to a tariff shock in that sector? We now turn to the final ingredient behind the transmission of GVC shocks to the capital-goods sector: high openness combined with a high elasticity of substitution between domestic and foreign intermediates. In the model, a sector's intermediate input demand reflects both how exposed it is to foreign inputs and how easily firms can substitute away from imported inputs when

Figure 4: Counterfactual with no Home Bias in Intermediate Inputs



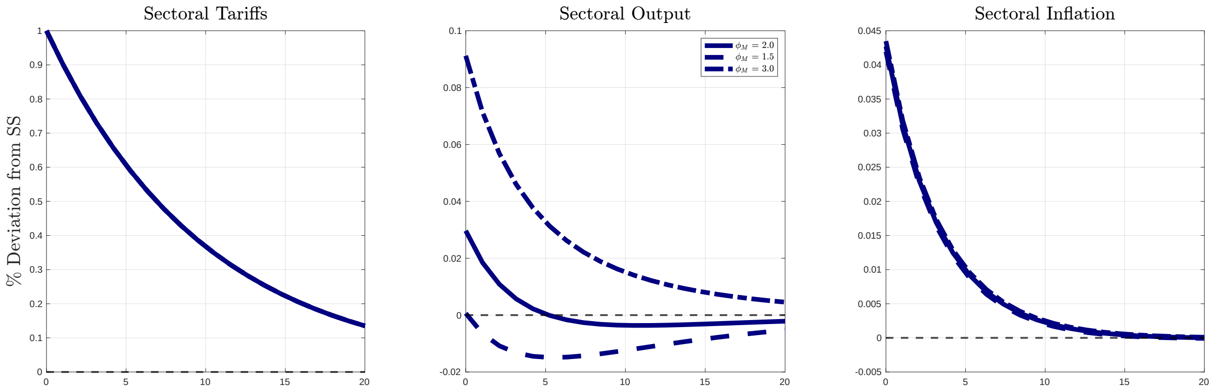
Notes: This figure shows the IRFs to a sectoral tariff shock. Row 1 shows the IRFs to a tariff shock in the capital-goods sectors. Row 2 shows the IRFs to a tariff shock in the manufacturing sector. Row 3 shows the IRFs to a tariff shock in the services sector.

their relative price goes up. The capital-goods sector is highly open in the benchmark calibration, and firms face an elasticity of substitution of 2 between domestic and foreign intermediates; a value sufficiently high to generate strong substitution effects.

To quantify this mechanism, Figure 5 reports the responses of the capital-goods sector to a GVC shock under three alternative elasticities of substitution: a lower value (1.5), our benchmark (2), and a higher value (3). The results confirm that substitutability is crucial for the sector's output response. When substitution channel is relatively dampened (elasticity of 1.5), the increase in the relative price of imported intermediates behaves like a standard cost push shock: marginal cost increases, inflation rises, and the sector's output falls. As the elasticity increases, firms shift more aggressively towards domestic intermediates when foreign inputs become more expensive. Because the capital-goods sector is very open, this substitution margin is quantitatively large. With the benchmark elasticity of 2, the substitution effect is strong enough to more than offset the direct cost push component, leading to an increase in sectoral output alongside rising inflation, the positive demand type pattern observed in the baseline. Thus, the combination of high openness and strong substitutability is what leads to the demand like expansion in the capital-goods sector in response to a de-integration shock.

When these ingredients are weakened, the sector's output falls while its inflation rises, highlighting that the amplification is not mechanical but structurally driven by the sector's position in global production networks. It is worth noting that, when we eliminate the home bias in the capital-goods sector, its output and inflation still increase as shown in Figure 4. The consumption of capital goods also has a very large import share; therefore, eliminating home bias only in production is not sufficient to eliminate the strong substitution effect that also arises in consumption.

Figure 5: Varying Elasticities of Substitution in Intermediate Inputs



Notes: This figure shows the IRFs to a tariff in the capital-goods sector under three alternative values of the elasticity of substitution between Home and Foreign intermediate inputs.

Our counterfactual exercises help explain why GVCs integration with EMEs matter for inflation dynamics. EME integration rose primarily in the capital-goods sectors, which are significantly more open than services and manufacturing. In such a sector, changes in the price or availability of foreign intermediates trigger a large reallocation of input demand and stronger movements in marginal cost. As a result, shocks to EME sourced inputs in capital goods transmit more forcefully to domestic inflation dynamics, often generating 'demand-like' output responses when substitutability between home and foreign inputs is high. This interaction of high openness and high substitutability gives capital-goods sectors a distinct impact on the inflation-output-gap relationship consistent with the pattern uncovered in our empirical analysis.

5 Conclusion

This paper examines how GVCs shape inflation dynamics, with a focus on the role of EMEs. We first show that imported intermediates enter marginal cost directly, weakening the sensitivity of inflation to domestic slack. A larger imported input share flattens the Phillips curve by reducing the weight of domestic inputs. Second, we show that terms-of-trade movements by transmitting international relative-price fluctuations into domestic inflation affect the inflation, output gap relationship. We then bring this prediction to the data and document that greater dependence on EME-sourced intermediates is associated with a significantly weaker relationship between inflation and the output gap across a range of reduced-form specifications. Crucially, it is not global integration per se, but integration with EMEs in particular, that underpins the empirical results.

To interpret this result, we develop a two-country model with trade in intermediates and sectoral input-output linkages. EMEs have become integrated predominantly through capital goods supply chains, rather than through services or manufacturing. Counterfactual exercises show that capital goods differ systematically from other sectors: they are substantially more open, source a wider set of foreign intermediates, and exhibit stronger input-demand reallocation when foreign intermediate prices shift. As a result, shocks to EME-sourced inputs propagate more powerfully through marginal costs in capital goods.

These findings highlight that the macroeconomic consequences of GVCs depend not only on the extent of integration but on where in the production network that integration occurs. Because

EME integration has been concentrated in capital goods, any prospective fragmentation or trade war will likewise have sector-specific consequences. Understanding these channels is central for evaluating how future changes in global production structures may shape inflation dynamics and the transmission of domestic policy.

References

AMBROSINO, L., J. CHAN, AND S. TENREYRO (2024): *Trade fragmentation, inflationary pressures and monetary policy*, Bank for International Settlements, Monetary and Economic Department.

AMITI, M., S. HEISE, F. KARAHAN, AND A. ŞAHIN (2023): "Inflation Strikes Back: The Role of Import Competition and the Labor Market," *NBER Macroeconomics Annual*, 38.

AUER, R. AND A. M. FISCHER (2010): "The effect of low-wage import competition on U.S. inflationary pressure," *Journal of Monetary Economics*, 57, 491–503.

AURAY, S., M. B. DEVEREUX, AND A. EYQUEM (2025): "Trade wars and the optimal design of monetary rules," *Journal of Monetary Economics*, 151, 103726.

AUTOR, D., D. DORN, AND G. H. HANSON (2013): "The China syndrome: Local labor market effects of import competition in the United States," *American Economic Review*, 103, 2121–68.

BALL, L. AND S. MAZUMDER (2019): "A Phillips curve with anchored expectations and short-term unemployment," *Journal of Money, Credit and Banking*, 51, 111–137.

BALL, L. M. (2006): "Has globalization changed inflation?" Tech. rep., National Bureau of Economic Research.

BERGIN, P. AND G. CORSETTI (2025): "Monetary Stabilization of Sectoral Tariffs," Working Paper 33845, National Bureau of Economic Research.

BERGIN, P. R. AND G. CORSETTI (2023): "The macroeconomic stabilization of tariff shocks: What is the optimal monetary response?" *Journal of International Economics*, 143, 103758.

BIANCHI, F. AND A. CIVELLI (2015): "Globalization and Inflation: Evidence from a Time-varying VAR," *Review of Economic Dynamics*, 18, 406–433.

BIANCHI, J. AND L. COULIBALY (2025): "The Optimal Monetary Policy Response to Tariffs," Working Paper 33560, National Bureau of Economic Research.

COMIN, D. A. AND R. C. JOHNSON (2020): "Offshoring and Inflation," NBER Working Papers 27957, National Bureau of Economic Research, Inc.

DE PAOLI, B. (2009): "Monetary Policy under Alternative Asset Market Structures: The Case of a Small Open Economy," *Journal of Money, Credit and Banking*, 41, 1301–1330.

DEVEREUX, M. B. AND C. ENGEL (2002): "Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect," *Journal of Monetary Economics*, 49, 913–940.

DI GIOVANNI, J., S. KALEMLI-ÖZCAN, A. SILVA, AND M. A. YILDIRIM (2023): "Pandemic-era inflation drivers and global spillovers," Tech. rep., National Bureau of Economic Research.

FEENSTRA, R. C. (1994): "New Product Varieties and the Measurement of International Prices," *American Economic Review*, 84, 157–177.

FORBES, K. (2019): "Inflation Dynamics: Dead, Dormant, or Determined Abroad?" Tech. rep., National Bureau of Economic Research.

GALÍ, J. AND T. MONACELLI (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 72, 707–734.

GILCHRIST, S. AND E. ZAKRAJSEK (2019): "Trade Exposure and the Evolution of Inflation Dynamics," Working Papers Central Bank of Chile 849, Central Bank of Chile.

GUERRIERI, L., C. GUST, AND J. D. LÓPEZ-SALIDO (2010): "International competition and inflation: a New Keynesian perspective," *American Economic Journal: Macroeconomics*, 2, 247–280.

GUILLOUX-NEFUSSI, S. (2020): "Globalization, market structure and inflation dynamics," *Journal of International Economics*, 123, 103292.

HEISE, S., F. KARAHAN, AND A. ŞAHIN (2022): "The Missing Inflation Puzzle: The Role of the Wage-Price Pass-Through," *Journal of Money, Credit and Banking*, 54, 7–51.

KALEMLI-ÖZCAN, S., C. SOYLU, AND M. A. YILDIRIM (2025): "Global Networks, Monetary Policy and Trade," Tech. rep., National Bureau of Economic Research.

MANKIW, N. G. AND R. REIS (2002): "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve," *The Quarterly Journal of Economics*, 117, 1295–1328.

MCLEAY, M. AND S. TENREYRO (2020): "Optimal inflation and the identification of the Phillips curve," *NBER Macroeconomics Annual*, 34, 199–255.

OBSTFELD, M. (2020): "Global Dimensions of U.S. Monetary Policy," *International Journal of Central Banking*, 16, 73–132.

RUBBO, E. (2020): "Networks, Phillips Curves and Monetary Policy," Tech. rep., mimeo, Harvard University.

——— (2023): "Networks, Phillips Curves, and Monetary Policy," *Econometrica*, 91, 1417–1455.

WERNING, I., G. LORENZONI, AND V. GUERRIERI (2025): "Tariffs as Cost-Push Shocks: Implications for Optimal Monetary Policy," Working Paper 33772, National Bureau of Economic Research.

Appendices

A Data Appendix

Sectoral Data: Sectoral inflation is calculated as a four-quarter percent change in Producer Price Index (PPI) and Service Producer Price Index (SPPI) from the Office of National Statistics (ONS). Data has been available at a quarterly frequency since 1997. The sectoral output series, Index of Production (IoP), and Index of Services (IoS) are also from ONS. Data has been available at a quarterly frequency since 1995 (1997 for the service sectors). Sectoral output gap series is calculated as the deviation indexes from their HP-filtered trends separately.

World Input-Output Database (WIOD): We use the 2016 version of the WIOD to calculate imports, exports, and imported intermediate good values for 56 sectors at an annual frequency from 2000 to 2014. However, the sectoral aggregation from WIOD does not match the aggregation level of sectoral price and output data from ONS. Therefore, we use many-to-many matching using the weights from the Blue Book GDP Source Catalogue.

Country Coverage: Following the IMF classification, we consider Brazil, Hungary, China, India, Indonesia, Mexico, Poland, Romania, Russia and Turkey as EMEs; Austria, Belgium, Czech Republic, Cyprus, Germany, Denmark, Spain, Estonia, Finland, France, Greece, Croatia, Hungary, Ireland, Italy, Lithuania, Latvia, Luxembourg, Malta, the Netherlands, Norway, Poland, Romania, Slovakia, Slovenia and Sweden as the EU and Australia, Canada, South Korea, Japan, U.S., Switzerland and the EU excluding Poland, Hungary and Romania as AEs.

Table A1: Sector Definitions

Sector	WIOT Industry Code	Industry Description
Capital Goods	C26-C30	Manufacture of electronics, electrical equipment, machinery, motor vehicles, other transport.
Manufacturing	C10-C25, C31-C33	Manufacture of food/beverage, textiles, wood and paper products, media, petroleum, chemicals, pharmaceuticals, rubber and plastics, metals, fabricated metals, mineral products, and repairs/installations.
Services	D35, E36-39, F, G45-G47, H49-H53, I, J58-J63, K64-K66, L68, M69-M75, N, O84, P85, Q, R, S, T, U	Utilities, Construction, Retail Trades, Transport, Postal, Hospitality, Media, Business and Tech, Miscellaneous Work, Public Administration, Defence, Education, etc.

B The Role of Trade

Here we explore the role of openness in the flattening of the UK's Phillips curve. We begin by displaying the trade openness and total import share over time (Figure A1) since the 1950s. Trade openness almost doubled from the mid-1980s to 2000 and then further increased by 50% from 2000 to 2020. Analogously, the share of imports doubled between the 1950s and 2010, remaining stable after that.

Both measures from Figure A1 point to a significantly increasing integration of the UK economy in global markets. We argue that increasing trade openness makes the prices in the UK economy less dependent on domestic factors. Therefore, the relationship between inflation and domestic economic activity weakens. To test this argument, we follow Ball (2006) and estimate the following regression where we interact aggregate output gap with trade openness

$$\pi_t = \beta_1 (y_t - y_t^*) + \beta_2 \text{Openness}_t + \beta_3 (y_t - y_t^*) \times \text{Openness}_t + \beta_4 \pi_t^M + \beta_5 \pi_t^{oil} + \beta_6 \left(\frac{1}{4} \sum_{j=1}^{4} \pi_{t-j} \right) + \varepsilon_t, \tag{B.1}$$

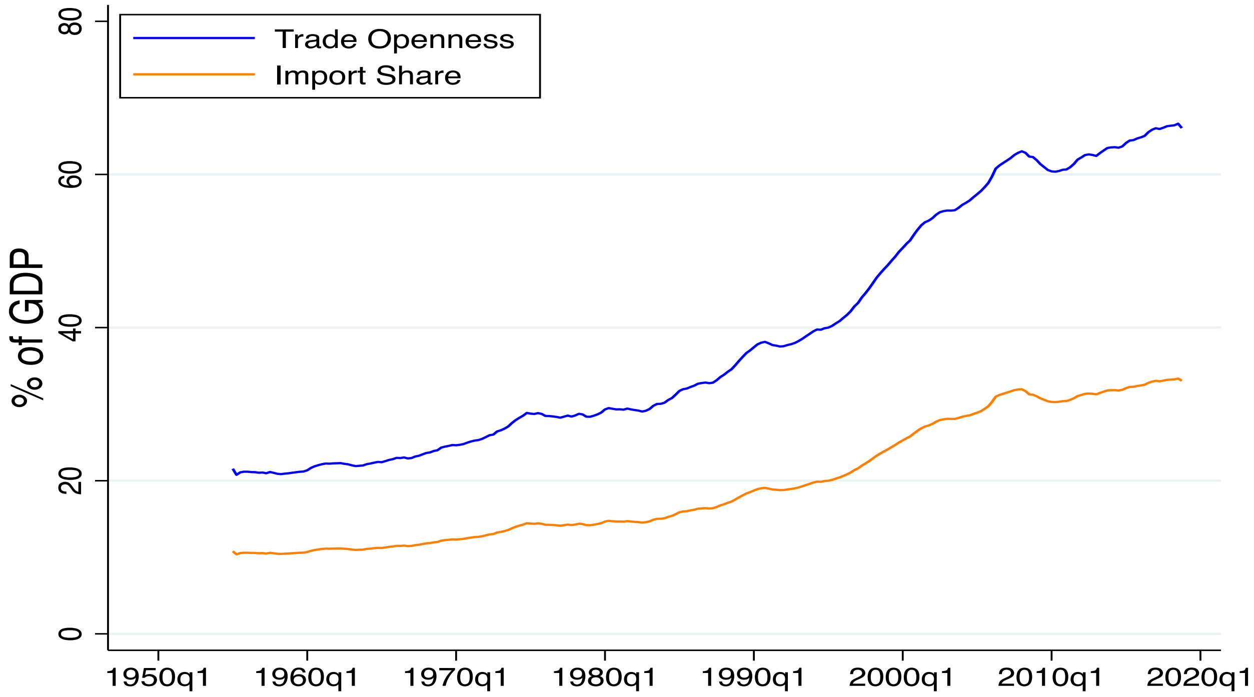
where $\text{Openness}_t = \frac{\text{Imports+Exports}}{\text{Real GDP}}$.[15] This variable is standardized (around the mean) to ease the interpretation of the estimated coefficients. We are interested in the estimation of the interaction parameter, β_3.

Table A2 column (1) suggests that the coefficients attached to $(y_t - y_t^*) \times \text{Openness}_t$ is negative and statistically significant, supporting the argument that rising trade openness in the UK led to a flattening in the Phillips curve. Recall that, Openness_t is standardized, thus β_1 coefficient denotes the Phillips curve slope for the mean trade openness period (e.g., the mid-1990s) in our sample and the coefficient for the interaction term (β_3) represents the effect of a one standard deviation increase in trade openness on the slope of the Phillips curve.

As a robustness check, we control the role of the inflation targeting regime in 1992 and central bank independence in 1997. We include a dummy variable equal to 1 after 1992 ($Post_{1992}$) and another one after 1997 ($Post_{1997}$) to control separately for the possible effects of these two policies. Columns (2)

[15] Aggregate price, output, imports and exports data are from the ONS. We calculate the aggregate output gap using the HP-filter.

Figure A1: Trade openness



Note: Trade Openness: $\frac{\text{Imports+Exports}}{\text{Real GDP}}$ and Import Share: $\frac{\text{Imports}}{\text{Real GDP}}$ using ONS data.

and (3) show that the results remain qualitatively unchanged, implying that one standard deviation increase in the trade variable flattens the slope of the Phillips curve to roughly 0.1.

The results imply that openness may be an important driver behind the flattening of the UK Phillips curve.

Table A2: Trade and the UK Phillips Curve (1980Q1-2017Q1)

π_t	(1)	(2)	(3)
$\widetilde{y}_t$	0.427***	0.426***	0.429***
	(0.0934)	(0.0936)	(0.0952)
Openness_t	0.00983	0.0804	0.238
	(0.0563)	(0.120)	(0.212)
$\widetilde{y}_t \times \text{Openness}_t$	-0.315***	-0.316***	-0.319***
	(0.0870)	(0.0883)	(0.0892)
π_t^{oil}	0.0104*	0.0104*	0.0107*
	(0.00593)	(0.00589)	(0.00588)
π_t^{M}	0.0498***	0.0500***	0.0489***
	(0.0179)	(0.0173)	(0.0176)
$\frac{1}{4}\sum_{j=1}^{4} \pi_{t-j}$	0.913***	0.918***	0.924***
	(0.0241)	(0.0274)	(0.0272)
No of Obs.	149	149	149
R^2	0.9674	0.9675	0.9677
$Post_{1992}$	No	Yes	No
$Post_{1997}$	No	No	Yes

Note: Newey-West standard errors in parentheses with a lag of 18.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

C Model Appendix

C.1 Derivation of Model Equations

Proof of Proposition 1. The marginal cost can be written as

$$\log MC = \delta \log W + (1-\delta) \log P^M - \log A,$$
$$\log MC^* = \delta^* \log W^* + (1-\delta^*) \log P^{M*} - \log A^*.$$

The input price index can be written as

$$\log P^M = \mu \log P_H + (1-\mu) \log P_F,$$
$$\log P^{M*} = \mu^* \log P_F^* + (1-\mu^*) \log P_H^*,$$

Combining the last two expressions yield

$$\log MC = \delta \log W + (1-\delta)(\mu \log P_H + (1-\mu) \log P_F) - \log A,$$
$$\log MC^* = \delta^* \log W^* + (1-\delta^*)(\mu \log P_F^* + (1-\mu) \log P_H^*) - \log A^*,$$

Under producer currency pricing, we have

$$\log P_F = \log P_F^* + \log \mathcal{E},$$
$$\log P_H = \log P_H^* + \log \mathcal{E},$$

where $\mathcal{E}$ is the nominal exchange rate (units of foreign currency in home currency). Plugging in PCP yields

$$\log MC = \delta \log W + (1-\delta)(\mu \log P_H + (1-\mu) \log P_F^* + (1-\mu) \log \mathcal{E}) - \log A,$$
$$\log MC^* = \delta \log W^* + (1-\delta)(\mu^* \log P_F^* + (1-\mu^*) \log P_H - (1-\mu) \log \mathcal{E}) - \log A^*.$$

In matrix notation, we can write the previous equation as

$$\log \mathbf{MC} = \boldsymbol{\delta} \cdot \log \mathbf{W} + \Omega \log \mathbf{p} + (\mathbf{1}-\boldsymbol{\delta}) \cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} \log \mathcal{E} - \log \mathbf{A}, \tag{C.1}$$

where $\log \boldsymbol{MC} = \begin{pmatrix} \log MC \\ \log MC^* \end{pmatrix}$. With nominal rigidities, domestic inflation is given by

$$d \log \mathbf{p} = \Theta \cdot d \log \mathbf{MC}, \tag{C.2}$$

where $\Theta = diag(1-\theta, 1-\theta^*)$. Plugging in this expression to a differenced version of (C.1), we get

$$d \log \mathbf{MC} = \boldsymbol{\delta} \cdot d \log \mathbf{W} + \Omega\Theta d \log \mathbf{MC} + (\mathbf{1}-\boldsymbol{\delta}) \cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} d \log \mathcal{E} - d \log \mathbf{A}.$$

Rearranging for marginal cost yields

$$d\log \mathbf{MC} = (1-\Omega\Theta)^{-1}\left(\delta \cdot d\log \mathbf{W} + (\mathbf{1}-\delta)\cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} d\log \mathcal{E} - d\log \mathbf{A}\right),$$

where the term $(I-\Omega\Theta)^{-1}$ captures the 'adjusted' Leontief inverse as in Rubbo (2023) - the production network structure of the economy, suitably adjusted for nominal rigidities. Plugging the previous equation into (C.2) yields

$$d\log \mathbf{p} = \Theta(1-\Omega\Theta)^{-1}\left(\delta \cdot d\log \mathbf{W} + (\mathbf{1}-\delta)\cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} d\log \mathcal{E} - d\log \mathbf{A}\right). \tag{C.3}$$

CPI inflation can be written as

$$d\log \mathbf{P} = \Phi d\log \mathbf{p} + \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E}, \tag{C.4}$$

where

$$\log \mathbf{P} = \begin{pmatrix} \log P \\ \log P^* \end{pmatrix}, \qquad \Phi = \begin{pmatrix} \alpha & 1-\alpha \\ 1-\alpha^* & \alpha^* \end{pmatrix}.$$

Market clearing. Now we write the Phillips Curve in terms of output gaps between home and foreign countries. Market clearing ensures that

$$Y_H = C_H + M_H + \frac{1-n}{n}\left(C_H^* + M_H^*\right), \tag{C.5}$$

$$Y_F^* = C_F^* + M_F^* + \frac{n}{1-n}\left(C_F + M_F\right). \tag{C.6}$$

We assume that there is balanced trade in final and intermediate goods [16]

$$nP_F(C_F + M_F) = (1-n)P_H(C_H^* + M_H^*), \tag{C.7}$$

and this allows us to write the market clearing (C.5) as

$$Y_H^{VA} \equiv Y_H - M_H - \frac{P_F}{P_H}M_F = C_H + \frac{P_F}{P_H}C_F, \tag{C.8}$$

where we define the value-added output as gross output less intermediate goods, both domestic and imported. We can then rewrite the previous equation to get the real consumption in terms of real value-added

$$P_H Y_H^{VA} = P_H C_H + P_F C_F = PC \Longleftrightarrow C = \frac{P_H}{P} Y_H^{VA}, \tag{C.9}$$

[16] Imposing this condition implies that the country size parameter no longer appears in the derivation below. However, the share parameters α and μ capture an equivalent notion.

Similarly, we can write foreign consumption in terms of foreign value-added

$$Y_F^{*VA} \equiv Y_F^* - M_F^* - \frac{P_H^*}{P_F^*} M_H^* = C_F^* + \frac{P_H^*}{P_F^*} C_H^*, \tag{C.10}$$

where the relative price follows from PCP. As above, we can rewrite the previous equation as

$$P_F^* Y_F^{*VA} = P_F^* C_F^* + P_H^* C_H^* = P^* C^* \iff C^* = \frac{P_F^*}{P^*} Y_F^{*VA}. \tag{C.11}$$

From the intra-temporal equation, we have

$$\begin{aligned} d\log W &= d\log P + \sigma d\log C + \varphi d\log L \\ &= \sigma d\log Y_H^{VA} + \varphi d\log L + \sigma(d\log P_H - d\log P), \end{aligned}$$

where the last equality follows (C.8). Similarly for the foreign economy,

$$d\log W^* - d\log P^* = \sigma d\log Y_F^{*VA} + \varphi d\log L^* + \sigma(d\log P_F^* - d\log P^*).$$

Now we write the previous two expressions in terms of the output gap. Using the definition of the output gap, we have

$$\begin{aligned} d\log W - d\log P &= \sigma(\tilde{y}_H + y_H^{nat}) + \varphi d\log L + \sigma(d\log P_H - d\log P) \\ &= \sigma\tilde{y}_H + \sigma y_H^{nat} + \varphi d\log L + \sigma(d\log P_H - d\log P). \end{aligned} \tag{C.12}$$

Part of the right-hand side is equal to

$$\begin{aligned} \sigma y_H^{nat} + \varphi d\log L &= \sigma y_H^{nat} + \varphi(d\log L - d\log L^{nat}) + \varphi d\log L^{nat} \\ &= \sigma y_H^{nat} + \varphi\tilde{y}_H + \varphi d\log L^{nat}, \end{aligned}$$

where the last equation follows from the equation $Y = AL$, since labor is the only factor of production. Continuing, we have

$$\begin{aligned} \sigma y_H^{nat} + \varphi d\log L &= \sigma(d\log L^{nat} + d\log A) + \varphi\tilde{y}_H + \varphi d\log L^{nat} \\ &= \varphi\tilde{y}_H + \sigma d\log A + (\sigma+\varphi) d\log L^{nat}. \end{aligned}$$

By Lemma 6 of Rubbo (2020)

$$d\log L^{nat} = \frac{1-\sigma}{\sigma+\varphi} d\log A, \tag{C.13}$$

hence

$$\begin{aligned} \sigma y_H^{nat} + \varphi d\log L &= \varphi\tilde{y}_H + \sigma d\log A + (\sigma+\varphi)\frac{1-\sigma}{\sigma+\varphi} d\log A \\ &= \varphi\tilde{y}_H + d\log A. \end{aligned} \tag{C.14}$$

Plugging the last equation into (C.12), we get

$$d\log W - d\log P + (\sigma+\varphi)\tilde{y}_H + d\log A + \sigma(d\log P_H - d\log P). \tag{C.15}$$

A similar expression can be derived for the foreign economy. Hence, in matrix form, we have

$$d\log \mathbf{W} - d\log \mathbf{P} = (\sigma+\varphi)\tilde{y} + d\log A + \sigma(d\log \mathbf{P} - d\log \mathbf{p}), \tag{C.16}$$

where $\tilde{y} = \begin{pmatrix} \tilde{y}_H \\ \tilde{y}_F^* \end{pmatrix}$. The last term of the previous equation is

$$\sigma(d\log \mathbf{P} - d\log \mathbf{p}) = \left(\sigma(I-\Phi)d\log \mathbf{p} - \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E}\right). \tag{C.17}$$

Hence we can rewrite (C.16) as

$$d\log \mathbf{W} - d\log \mathbf{P} = (\sigma+\varphi)\tilde{y} + d\log \mathbf{A} + \sigma(I-\Phi)d\log \mathbf{p} - \sigma \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E}. \tag{C.18}$$

Using (C.4), we can also write

$$d\log \mathbf{W} - d\log \mathbf{P} = d\log \mathbf{W} - \Phi d\log \mathbf{p} - \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E}. \tag{C.19}$$

Plug in for $d\log \mathbf{p}$ using (C.3), we get

$$\begin{aligned} d\log \mathbf{W} - d\log \mathbf{P} = d\log \mathbf{W} &- \Phi\Theta(I-\Omega\Theta)^{-1} \\ &\left[\delta \cdot d\log \mathbf{W} + (1-\delta)\cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} d\log \mathcal{E} - d\log \mathbf{A}\right] \\ &- \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E}. \end{aligned} \tag{C.20}$$

Expand and collect

$$\begin{aligned} d\log \mathbf{W} - d\log \mathbf{P} = &\left[I - \Phi\Theta(I-\Omega\Theta)^{-1}\delta\right] d\log \mathbf{W} + \Phi\Theta(I-\Omega\Theta)^{-1} d\log \mathbf{A} \\ &- \left[\Phi\Omega(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} + \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix}\right] d\log \mathcal{E}. \end{aligned} \tag{C.21}$$

Combine (C.18) and (C.21)

$$\begin{aligned} (\sigma+\varphi)\tilde{y} + \sigma(I-\Phi)d\log \mathbf{p} - \sigma \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix} d\log \mathcal{E} = &\left[I - \Phi\Theta(I-\Omega\Theta)^{-1}\delta\right] d\log \mathbf{W} \\ &+ \Phi\Theta(I-\Omega\Theta)^{-1} d\log \mathbf{A} \\ &- \left[\Phi\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot \begin{pmatrix} 1-\mu \\ -(1-\mu^*) \end{pmatrix} + \begin{pmatrix} 1-\alpha \\ -(1-\alpha^*) \end{pmatrix}\right] d\log \mathcal{E}. \end{aligned} \tag{C.22}$$

Collect terms

$$
\begin{aligned}
&(\sigma+\varphi)\widetilde{y}+\left[I-\Phi\Theta(I-\Omega\Theta)^{-1}\right]d\log\mathbf{A}\\
&+\left[\Phi\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right]d\log\mathcal{E}\\
&+\sigma(I-\Phi)d\log\mathbf{p}=[I-\Phi\Theta(I-\Omega\Theta)^{-1}\delta]d\log\mathbf{W}.
\end{aligned}
\tag{C.23}
$$

Plug in for $d\log\mathbf{p}$ using (C.3)

$$
\begin{aligned}
&(\sigma+\varphi)\widetilde{y}+[I-\Phi\Theta(I-\Omega\Theta)^{-1}]d\log\mathbf{A}\\
&+\left[\Phi\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right.\\
&\left.+\sigma(I-\Phi)\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}\right)d\log\mathcal{E}\\
&=\left[I-\Phi\Theta(I-\Omega\Theta)^{-1}\delta-\sigma(I-\Phi)\Theta(I-\Omega\Theta)^{-1}\delta\right]d\log\mathbf{W}.
\end{aligned}
\tag{C.24}
$$

Collect terms

$$
\begin{aligned}
&(\sigma+\varphi)\widetilde{y}+\left[I-\Phi\Theta(I-\Omega\Theta)^{-1}-\sigma(I-\Phi)\Theta(I-\Omega\Theta)\right]d\log\mathbf{A}\\
&+\left[\Phi\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right.\\
&\left.+\sigma(I-\Phi)\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}\right)d\log\mathcal{E}\\
&=\left[I-\Phi\Theta(I-\Omega\Theta)^{-1}\delta-\sigma(I-\Phi)\Theta(I-\Omega\Theta)^{-1}\delta\right]d\log\mathbf{W}.
\end{aligned}
\tag{C.25}
$$

Simplify

$$
\begin{aligned}
&(\sigma+\varphi)\widetilde{y}+\left[I-((1-\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}\right]d\log\mathbf{A}\\
&+\left[((1-\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right]d\log\mathcal{E}\\
&=[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta]d\log\mathbf{W}.
\end{aligned}
\tag{C.26}
$$

Rearrange for $d\log\mathbf{W}$

$$
\begin{aligned}
&d\log\mathbf{W}=[I-((1+\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}\delta]^{-1}\\
&[(\sigma+\varphi)\widetilde{y}+[I-[(1-\sigma)I+\sigma\Phi]\Theta(I-\Omega\Theta)^{-1}]d\log\bar{A}\\
&+\left[((1-\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right]d\log\mathcal{E}.
\end{aligned}
\tag{C.27}
$$

Plug back into (C.3)

$$
\begin{aligned}
d\log \mathbf{p} = \Theta(I-\Omega\Theta)^{-1}\delta \Bigg(& [I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta]^{-1} \\
& \left\{ (\sigma+\varphi)\widetilde{y} + \left[I-((1-\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}\right] d\log \mathbf{A} \right. \\
+ \left[((1-\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(1-\delta)\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix} + (1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right] & \left. d\log \mathcal{E} \right\} \Bigg) \\
+\Theta(I-\Omega\Theta)^{-1} & \left[(\mathbf{1}-\boldsymbol{\delta})\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix} d\log\mathcal{E} - d\log \mathbf{A}\right].
\end{aligned} \tag{C.28}
$$

Collect terms

$$
\begin{aligned}
& d\log \mathbf{p} = \Theta(I-\Omega\Theta)^{-1}\boldsymbol{\delta} \\
& \left[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta\right]^{-1}(\sigma+\varphi)\widetilde{y} \\
& + \Bigg[\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Omega(I-\Omega\Theta)^{-1}\delta]^{-1}[I-((1-\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}] \\
& - \Theta(I-\Omega\Theta)^{-1}\Bigg] d\log \mathbf{A} \\
& + \Bigg[\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta]^{-1} \\
& \left\{((1+\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(1-\delta)\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix} + (1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right\} \\
& + \Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\boldsymbol{\delta})\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}\Bigg] d\log\mathcal{E}.
\end{aligned} \tag{C.29}
$$

Use (C.4) to get CPI Phillips Curves

$$
\begin{aligned}
d\log \mathbf{P} &= \Phi\Theta(I-\Omega\Theta)^{-1}\delta\left[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta\right]^{-1}(\sigma+\varphi)\widetilde{y} \\
&+\Phi\Bigg[\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Omega(I-\Omega\Theta)^{-1}\delta]^{-1}[I-((1-\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}] \\
&-\Theta(I-\Omega\Theta)^{-1}\Bigg]d\log \mathbf{A} \\
&+\Phi\Bigg[\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta]^{-1} \\
&\left\{((1+\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right\} \\
&+\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\Bigg]d\log\mathcal{E},
\end{aligned}
\tag{C.30}
$$

where

$$
\mathcal{K}=\Phi\Theta(I-\Omega\Theta)^{-1}\delta\left[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta\right]^{-1}(\sigma+\varphi),
$$

$$
\begin{aligned}
\mathcal{G}=\Phi\Bigg[&\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Omega(I-\Omega\Theta)^{-1}\delta]^{-1} \\
&[I-((1-\sigma)I+\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}]-\Theta(I-\Omega\Theta)^{-1}\Bigg],
\end{aligned}
$$

and

$$
\begin{aligned}
\mathcal{H}=\Phi\Bigg[&\Theta(I-\Omega\Theta)^{-1}\delta[I-((1+\sigma)\Phi-\sigma I)\Theta(I-\Omega\Theta)^{-1}\delta]^{-1} \\
&\left\{((1+\sigma)I-\sigma\Phi)\Theta(I-\Omega\Theta)^{-1}(1-\delta)\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+(1-\sigma)\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\right\} \\
&+\Theta(I-\Omega\Theta)^{-1}(\mathbf{1}-\delta)\cdot\begin{pmatrix}1-\mu\\-(1-\mu^*)\end{pmatrix}+\begin{pmatrix}1-\alpha\\-(1-\alpha^*)\end{pmatrix}\Bigg].
\end{aligned}
$$